Revision of the GUM: why and how?

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Why a revision?
Merits of the GUM

• Provides widely accepted guidance on measurement uncertainty

• Treats in a common way systematic and random contributions

• Rests on solid principles of probability and statistics

• It is accused of being difficult (by some) or simplistic (by others), which means that it is a good compromise
• First publication in 1993

• Reprint in 1995 with some corrections

• JCGM 100:2008 (free of charge) GUM 1995 with minor modifications

• Until now, a large number of documents based on the GUM has been written. The GUM has been translated into many languages

• In addition, the GUM has been adopted as a standard, in some cases as a law, in many countries
### GUM-translations since 2008

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Courtesy BIPM
### CEN On-line catalogue

**ICS:** 17.020 - Metrology and measurement in general  
**Reference number:** ENV 13005:1999  
**Title:** Guide to the expression of uncertainty in measurement

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On these grounds, a revision of the GUM needs careful consideration and strong motivation
Drawbacks of the GUM

**It is a compromise**

- It is difficult
- It is simplistic
The GUM is difficult

- **Its application** requires notions of
  - Calculus (partial derivatives)
  - Probability (densities and their moments, mean and variance)
  - Statistics (sample statistics, average and standard deviation)

- **Its understanding** requires solid background in
  - Theory of measurement (concepts such as quantity, error, model)
  - Probability and statistics (random variables, differing views of probability, central limit theorem, convolutions, several distributions)
The GUM is simplistic

- No guidance on the (frequent) case of many measurands

- Poor guidance on the construction of a coverage interval (emphasis is on standard uncertainty), limited to a situation optimistically considered as frequently occurring

- Other (comparatively minor) weak sides, such as poor consideration to
  - non-symmetric distributions
  - non-linear measurement models

- The cases above are difficult, probably they had not been considered in the first edition on purpose
Remedies to difficulty

• NONE. *Things should be made as simple as possible, but not simpler*

• The GUM is and will remain a high-level document, some difficulty is unavoidable

• However, the next GUM will be at a level comparable to that of the current GUM – still based on a first-order expansion or, ultimately, Gauss’ law of errors
Are the cases not covered in the GUM of practical importance?

- Any calibration of a set of artefacts, be they weights, capacitors, gauge blocks or similar, is a multivariate case.

- The CIPM MRA asks for CMCs at the 95 % coverage probability, i.e., CMCs are coverage intervals.

- Not a few quantities of practical importance are such that the current practice $U = ku$ (with typically $k = 2$) is inappropriate.
There was a real need to address the cases not covered in the current GUM
Remedies to simplism

- Difficult problems typically imply difficult solutions
- Coverage interval (and more): see JCGM 101:2008 (Supplement 1)
- Multivariate case: see JCGM 102:2011 (Supplement 2)
Both problems (multivariate case and coverage interval) and solutions were kept out of the GUM, in the attempt to avoid a deep revision.
Side effects of remedies

- The GUM and its Supplements are now inconsistent

Why didn’t we write Supplements consistent with the GUM?
The GUM is ambiguous

The definition of uncertainty in the GUM is

parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand

This is an intrinsically Bayesian view of uncertainty – uncertainty concerns the measurand

The definition contrasts with the way in which uncertainty is obtained, essentially frequentist – uncertainty concerns the measurand estimate, and is itself uncertain
Supplements are unambiguous

- In Supplement 1, PDFs (probability density functions) are used to describe the state of knowledge about each input quantity.

- Accordingly, the state of knowledge about the measurand is described by a PDF obtained from those of the input quantities through the measurement model (in a way that is not relevant here).

- This is an intrinsically Bayesian attitude, and is consistently adopted throughout the Supplements.

No alternative was possible!
How to revise the GUM?

• Main purpose: to make it consistent with its Supplements

• Secondary purposes:
  • to make it consistent as much as possible with VIM3
  • to broaden its applicability to “new” needs
  • to minimize notational and terminological ambiguities
Alignment with Supplements

• Uncertainties (and estimates) are:
  
  – estimates of moments of frequency distributions, in the current GUM (they have degrees of freedom)
  
  – exact moments of state-of-knowledge distributions, in the Supplements (no degrees of freedom)

• In the revised GUM, uncertainties (and estimates) will be exact moments of state-of-knowledge distributions, as in the Supplements
Practical impact on standard uncertainty

- With respect to the current GUM, input standard uncertainties obtained from a sample of \( n > 3 \) repeated indications will be larger by a factor \( \sqrt{(n - 1)/(n - 3)} \)

- As a consequence, the output standard uncertainty, *ceteris paribus*, will change, being anyway consistent with the (uncertain) uncertainty provided by the current GUM

- Classification into Type A and Type B evaluations loses its scientific basis – will be kept (de-emphasized) due to non-scientific considerations

- No longer effective degrees of freedom attached to the output uncertainty - Welch-Satterthwaite formula no longer needed
Practical impact on coverage intervals

• In the revised GUM there will be mostly generic guidance on the construction of coverage intervals, this task being given to Supplement 1

• Distribution-free coverage intervals, based on Chebyshev or Gauss inequalities, will be given

• Expanded uncertainty de-emphasized

• Greater consideration to non-symmetric coverage intervals

• Possible impact on KCDB, Appendix C
Cosmetic changes

• Suffix «c» in the combined standard uncertainty \( u_c \) dropped (as in JCGM 101, JCGM 102 and JCGM 106)

• New notation \( u_x \) allowed as an alternative to \( u(x) \)

• Introduction of the hatted symbol \( \hat{T} \), say, for the estimate of a temperature \( T \) (when appropriate)

• Introduction of matrix notation, in parallel with, not in substitution of conventional notation
Further notable features

• Increased guidance on the evaluation of input uncertainties

• Guidance on the evaluation of input covariances

• Clarification of the meaning of loose expressions such as «uncertainty of…» through a dedicated section

• Enhanced examples. Examples concerning the GUM and its Supplements will be collected in a separate document
Thank you for your attention