TÉCNICO LISBOA

instituto de telecomunicações

Electrical Impedance Measurements:
Algorithms for impedance estimation and equivalent circuit parameters determination

Pedro M. Ramos
pedro.m.ramos@tecnico.ulisboa.pt

TÉCNICO LISBOA

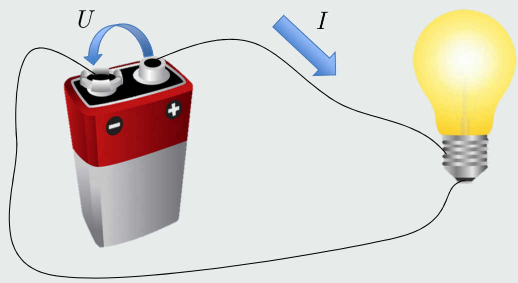
instituto de telecomunicações

Introduction

- What is electrical impedance?
- Why does it matter?
- How do we measure electrical impedance?
- How do we know the circuit of the measured impedance?

What is electrical impedance?

- In DC (direct current), the relation between the voltage and the current applied to a conductor, is the electrical resistance.



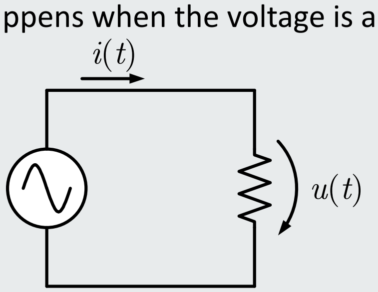
$$I = \frac{U}{R}$$

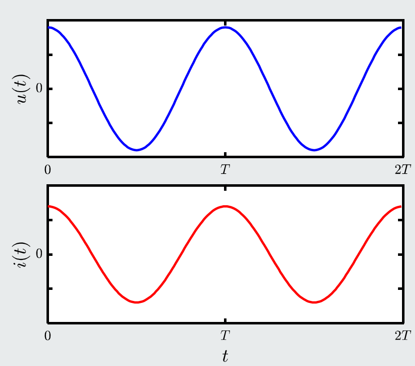
$$R = \frac{U}{I}$$

- This is Ohm's law.

What is electrical impedance?

- What happens when the voltage is a sinewave AC (from alternate current)?



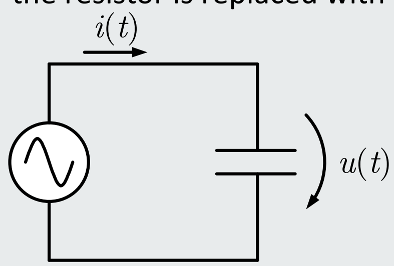


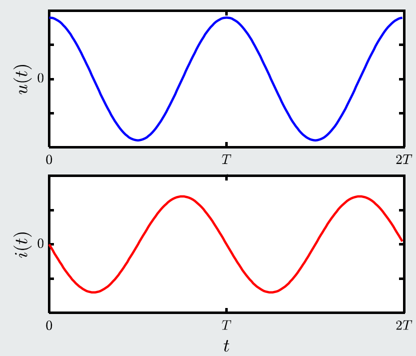
$$R = \frac{U_{RMS}}{I_{RMS}}$$

$$U_{RMS} = \sqrt{\frac{1}{T}\int_T u^2(t)\,dt}$$

What is electrical impedance?

- What if the resistor is replaced with a capacitor?





$$\frac{u(t)}{i(t)} = ?$$

What is electrical impedance?

- With sinewaves, it is better not to use $u(t)$ and $i(t)$ and use instead phasors.
- These are complex numbers that fully describe the sinewave

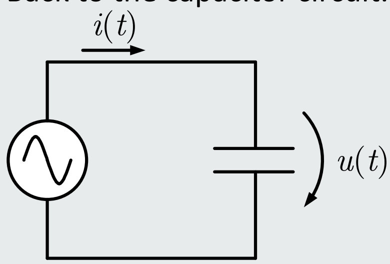
$$u(t) = U\cos\left(2\pi f t + \varphi_U\right) \Longrightarrow \overline{U} = U\,e^{j\varphi_U}$$

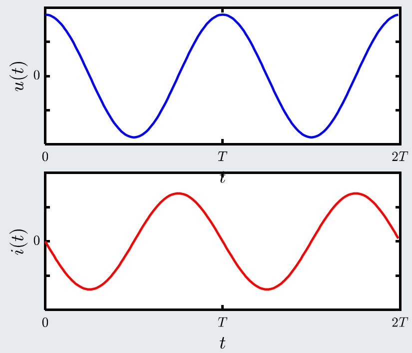
$$i(t) = I\cos\left(2\pi f t + \varphi_I\right) \Longrightarrow \overline{I} = I\,e^{j\varphi_I}$$

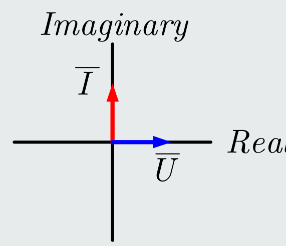
- The frequency f is implicit when phasors are used ($\omega = 2\pi f$).
- f is needed to go back from phasors to $u(t)$ and $i(t)$.

What is electrical impedance?

- Back to the capacitor circuit. How/where are the phasors?

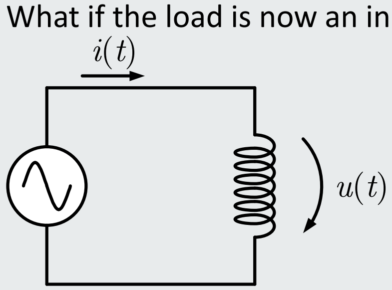


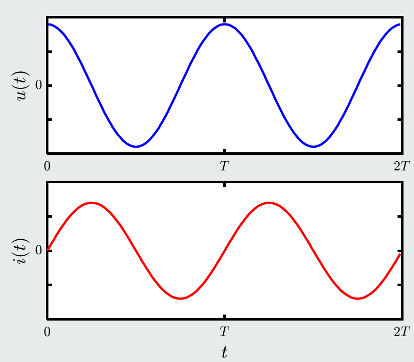


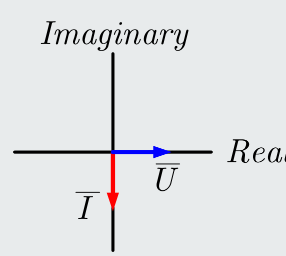


What is electrical impedance?

- What if the load is now an inductor?







TÉCNICO LISBOA

instituto de telecomunicações

Gim Instrumentation and Measurement Group

What is electrical impedance?

- Electrical impedance can be defined with sinewave stimulus using the phasors

$$\overline{U} = U\ e^{j\varphi_U}$$

$$\overline{I} = I\ e^{j\varphi_I}$$

- The electrical impedance Z is

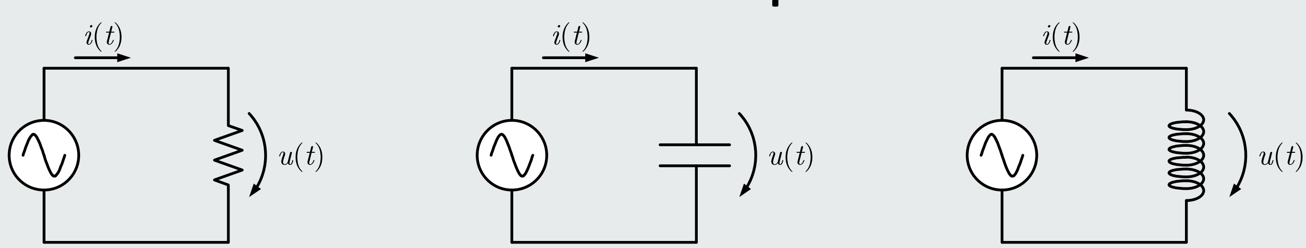
$$Z = \frac{\overline{U}}{\overline{I}} = \frac{U}{I}\ e^{j(\varphi_U - \varphi_I)} = |Z|\ e^{j\varphi_Z}$$

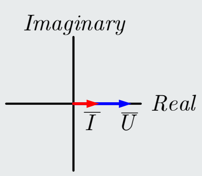
TÉCNICO LISBOA

instituto de telecomunicações

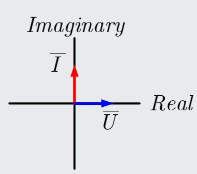
Gim Instrumentation and Measurement Group

What is electrical impedance?

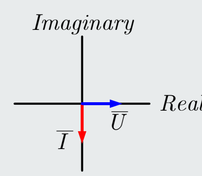




$$Z = R$$
$$|Z| = R$$
$$\varphi_Z = 0$$



$$Z = \frac{1}{j\omega C}$$
$$|Z| = \frac{1}{\omega C}$$
$$\varphi_Z = -\pi / 2$$

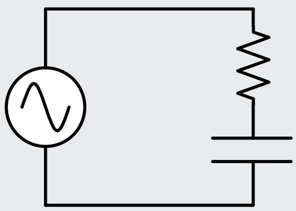


$$Z = j\omega L$$
$$|Z| = \omega L$$
$$\varphi_Z = \pi / 2$$

What is electrical impedance?

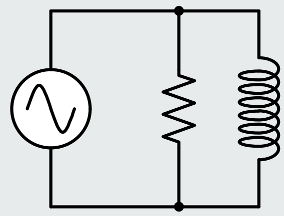
- These basic elements can be combined. For example:

In series



$$Z = Z_1 + Z_2 = R + \frac{1}{j\omega C}$$

In parallel



$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{j\omega RL}{R + j\omega L}$$

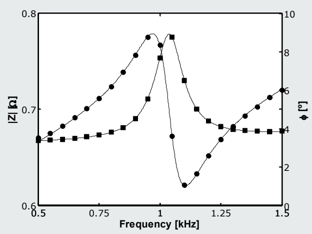
Why does it matter?

- Some examples:
  - Electrical impedance is used in many sensors where the measurand changes the impedance frequency response of the sensor.
  - Bioimpedance is used in non-invasive test monitoring of living organisms.
  - In rechargeable battery systems, the battery output impedance can be used to estimate the battery SOH (state of health).

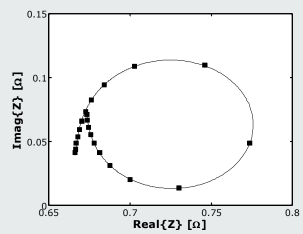
Why does it matter?

- To derive, useful information from the impedance, it must be measured at different frequencies. Example, for a vibrating wire viscosity sensor:

Impedance frequency response



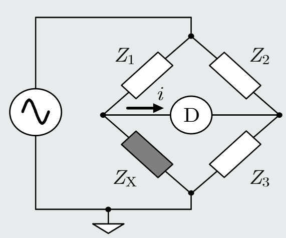
Nyquist Plot



- Impedance Spectroscopy (IS)

How is electrical impedance measured?

- Pre-digital, impedance measurement methods included mostly bridges:

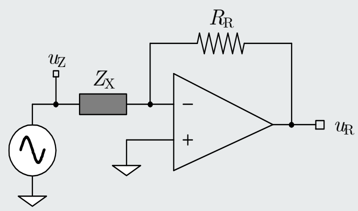


- Manual adjustment of Z_1, Z_2 and Z_3 to cancel i, leads to $Z_x = Z_1 Z_3 / Z_2$.
- High level of expertise and experience required.
- Measuring many, many frequencies is almost impossible.

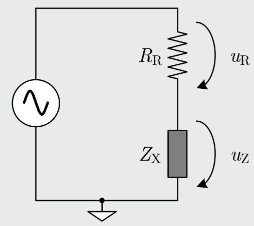
How is electrical impedance measured?

- Evolution of analog electronics, ADCs and digital signal processors, have led to the development of highly efficient impedance measurement devices.

Auto-balancing bridge method



Voltage-current method

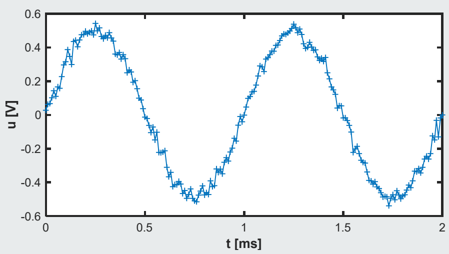


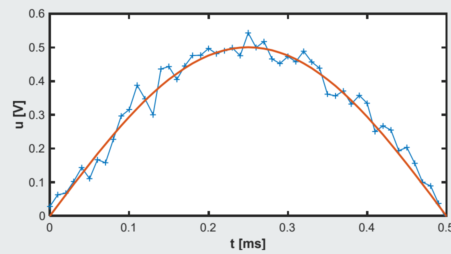
How is electrical impedance measured?

- In these two measurement setups, ADCs acquire samples from two sinewave voltages.
- Algorithms estimate the sinewave parameters and from there, the impedance parameters ($|Z|$ and φ_Z) are estimated.
- The efficiency of these algorithms is crucial. This includes their uncertainty and also how long does the calculation require.

How is electrical impedance measured?

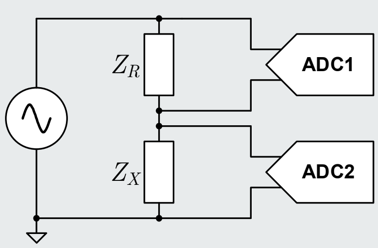
- One algorithm that we have extensively used is sine-fitting.
- It estimates the sinewave parameters (amplitude, phase, DC component and frequency) of a set of acquired samples from a sinewave voltage.

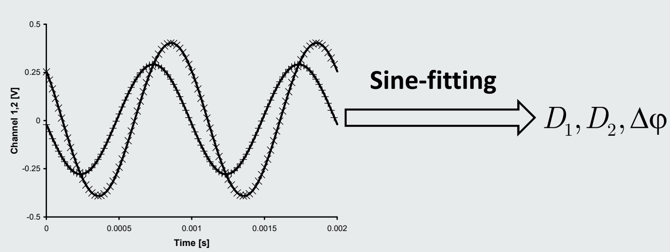




How is electrical impedance measured?

- Measurement setup





$$|Z_X| = \frac{D_2}{D_1}|Z_R| \qquad \varphi_Z = \Delta\varphi + \varphi_{Z_R}$$

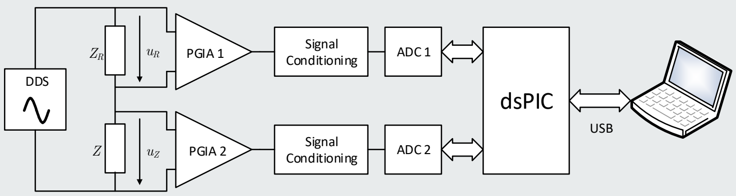
TÉCNICO LISBOA

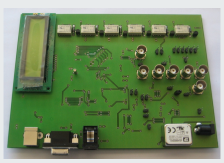
instituto de telecomunicações

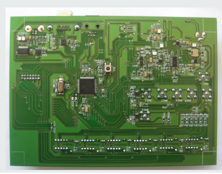
Gim Instrumentation and Measurement Group

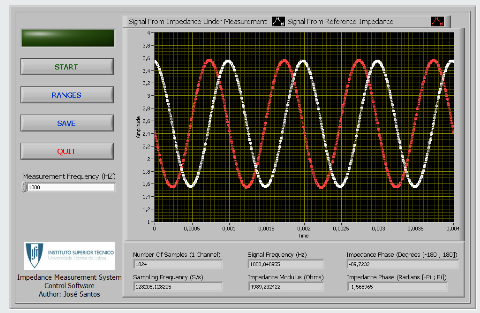
How is electrical impedance measured?

- Implemented in a dsPIC based embedded measurement system.









TÉCNICO LISBOA

instituto de telecomunicações

Gim Instrumentation and Measurement Group

How is electrical impedance measured?

- How to measure the impedance frequency response?
- **Method A: Single-tone sweep method**
  1. Set the measurement frequency on the generator;
  2. Acquire samples from both ADCs;
  3. Use sine-fitting to estimate sinewave voltage parameters and from them, the impedance parameters (for that measurement frequency);
  4. Repeat steps 1-3 for the next measurement frequencies.

TÉCNICO LISBOA

instituto de telecomunicações

Instrumentation and Measurement Group

How is electrical impedance measured?

- How to measure the impedance frequency response?
- **Method B: Multi-harmonic method**
  1. Design multi-harmonic stimulus;
  2. Upload stimulus to arbitrary waveform generator (AWG);
  3. Acquire samples from both ADCs;
  4. Use multi-harmonic waveform fitting algorithm to estimate harmonic amplitude and phases of the acquired voltages and from them, the impedance parameters (at the frequencies of all harmonics).

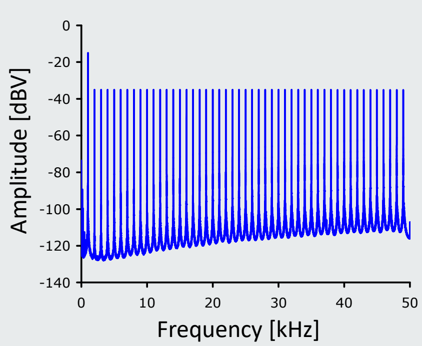
TÉCNICO LISBOA

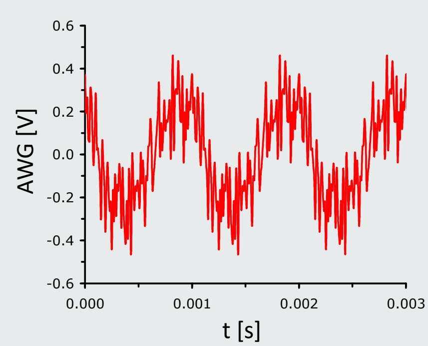
instituto de telecomunicações

Instrumentation and Measurement Group

How is electrical impedance measured?

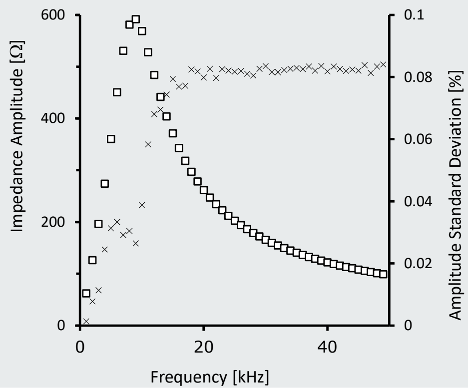
- Example 1: $f = 1$ kHz, with higher fundamental and 48 lower amplitude harmonics

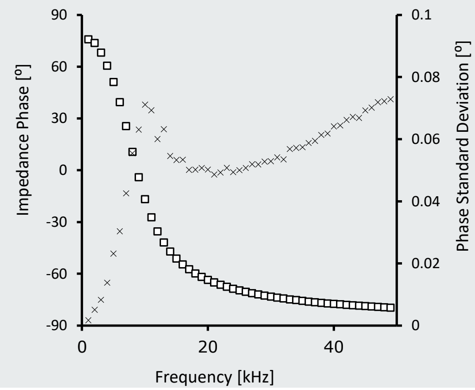




How is electrical impedance measured?

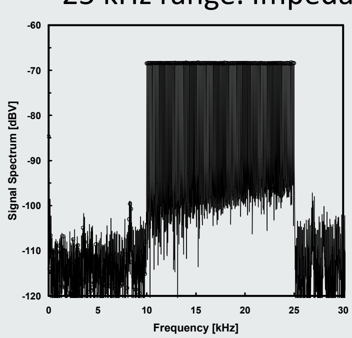
- Results from example 1 with RLC parallel circuit with resonance near 9 kHz.

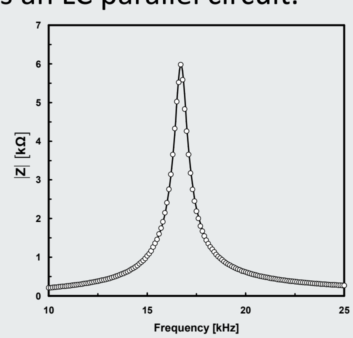


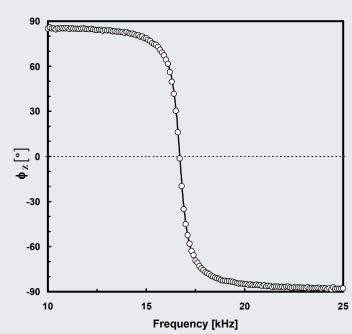


How is electrical impedance measured?

- Example 2: f = 100 Hz, without fundamental and 250 harmonics in the 10 kHz to 25 kHz range. Impedance is an LC parallel circuit.







TÉCNICO LISBOA

instituto de telecomunicações

Gim Instrumentation and Measurement Group

How do we know the impedance circuit?

- This is split into two parts:
  - Part 1: If the circuit topology is known, how do we estimate the circuit parameters?
  - Part 2: If the circuit topology is unknown, how do we determine the circuit topology?

TÉCNICO LISBOA

instituto de telecomunicações

Gim Instrumentation and Measurement Group

How to estimate the circuit parameters?

- Option 1: CNLS (Complex Nonlinear Least-Squares)
  - Developed by J. R. Macdonald.
  - It is a algorithm that, given a circuit topology, estimates the values of the components that best fit with the measured IFR.
  - It is a closed program with multiple circuit topologies.
  - Highly configurable but requires good initial estimates to converge to the correct circuit parameters.

How to estimate the circuit parameters?

- Option 2: Genetic Algorithms (GAs)
  - Genetic algorithms can efficiently search a vast, multidimensional search space and find the approximate location of the absolute minimum.
  - To effectively search the multi-decade search space (associated with component values), the estimated value is the log 10 base value of the component value.
  - Requires many iterations and fine-tuning.

How to identify the impedance circuit?

- Option 1: Vector Fitting (VF)

  Estimates the coefficients of two polynomials whose ratio fits the measured Impedance Frequency Response (IFR).

  It does not estimate directly the circuit topology.
- Option 2: Gene Expression Programming (GEP)

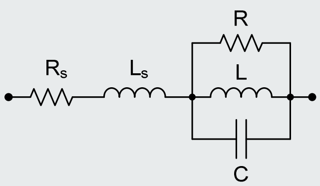
  Evolutionary algorithm specifically used to obtain the circuit topology that best fits the measured IFR.

How to identify the impedance circuit?

- Gene Expression Programming (GEP)

  The circuit topology is described in a tree structure.

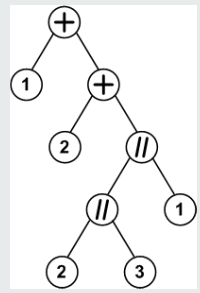
  Example:



Operators

R ⟶ 1
L ⟶ 2
C ⟶ 3

Series ⟶ +
Parallel ⟶ //



Gene	+	1	+	2	//	//	1	2	3
		R		L			R	L	C

How to identify the impedance circuit?

- Gene Expression Programming (GEP)
  - A set of circuits (population) is evaluated to assess its fitness.
  - Each circuit is described by a Gene which is a sequence of elements.
  - There is a maximum sequence size.
  - To ensure that the sequence corresponds to a valid circuit, it has a head and a tail. The head has operators and components while the tail only has components.
  - GEP operations combine and change these sequences.

TÉCNICO LISBOA

instituto de telecomunicações

How to identify the impedance circuit?

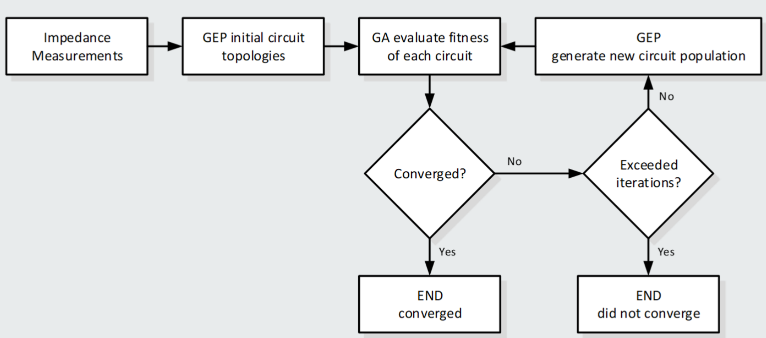
- Gene Expression Programming (GEP) Operations
  - Replication: a new population of circuits is obtained from the previous one based on their fitness.
  - Mutation: some random positions on a few random genes are changed.
  - Transposition: parts of the gene are copied to another location within the gene.
  - Recombination: pairs of randomly chosen genes exchange part of their gene code.
  - In mutation and transposition, care must be taken to ensure that GEP coding rules are maintained.
  - The best gene is always carried to next generation (survival of the fittest).

TÉCNICO LISBOA

instituto de telecomunicações

How to identify the impedance circuit?

- Gene Expression Programming (GEP)

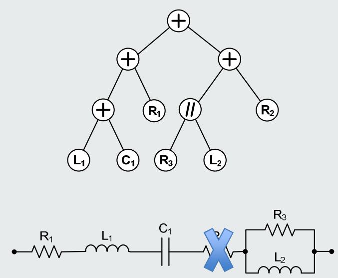


TÉCNICO LISBOA

instituto de telecomunicações

How to identify the impedance circuit?

- Gene Expression Programming (GEP)
  - Simplification routine identifies components that can be removed from the circuit:



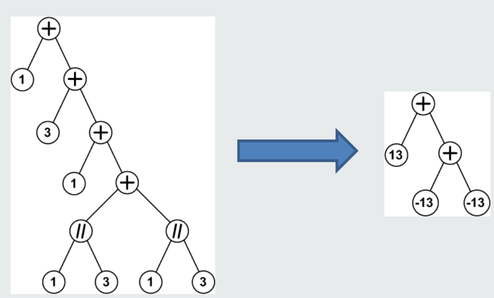
TÉCNICO LISBOA

instituto de telecomunicações

How to identify the impedance circuit?

- Gene Expression Programming (GEP)
  - Using more complex circuit elements:

Series Combinations		Parallel Combinations	
Code	Type	Code	Type
12	R+L	-12	R//L
13	R+C	-13	R//C
23	L+C	-23	L//C
123	R+L+C	-123	R//L//C



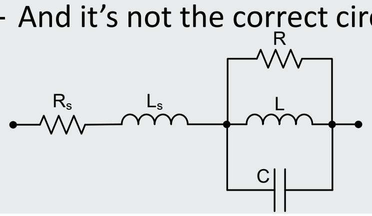
TÉCNICO LISBOA

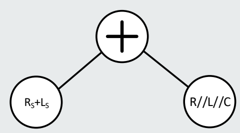
instituto de telecomunicações

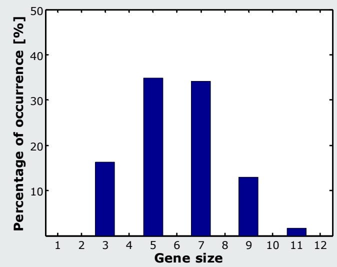
Gim Instrumentation and Measurement Group

How to identify the impedance circuit?

- Gene Expression Programming (GEP)
  - In most cases, with convergence, additional components are added to the gene.
  - These do not affect the impedance response at the measured frequency.
  - However, they appear as longer genes.
  - And it's not the correct circuit!







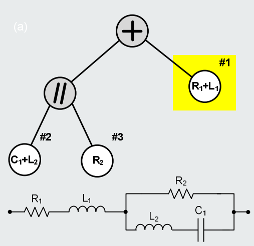
TÉCNICO LISBOA

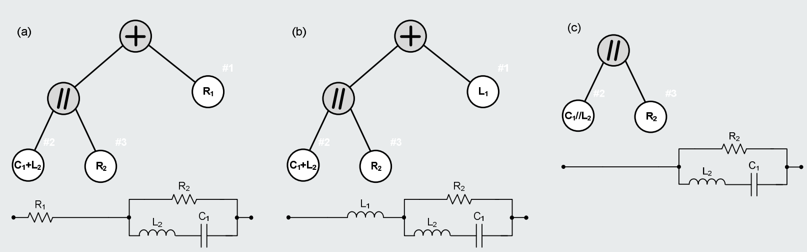
instituto de telecomunicações

Gim Instrumentation and Measurement Group

How to identify the impedance circuit?

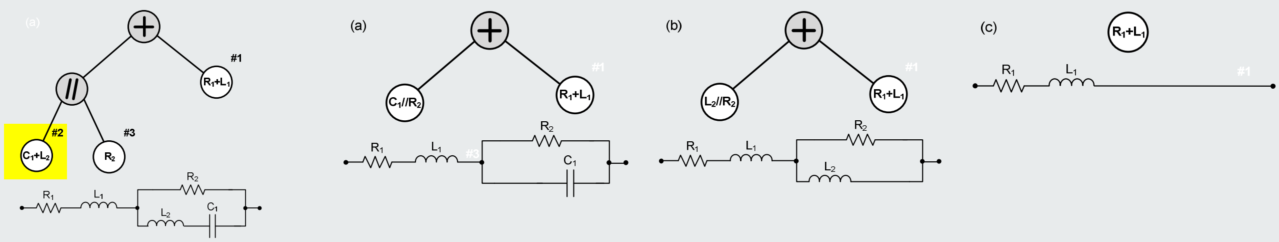
- Gene Expression Programming (GEP)
  - How to remove components that are not needed?





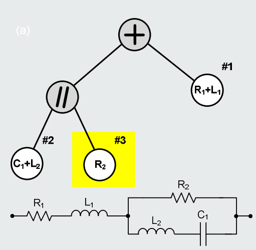
How to identify the impedance circuit?

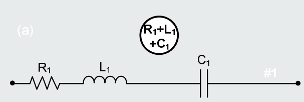
- Gene Expression Programming (GEP)
  - How to remove components that are not needed?



How to identify the impedance circuit?

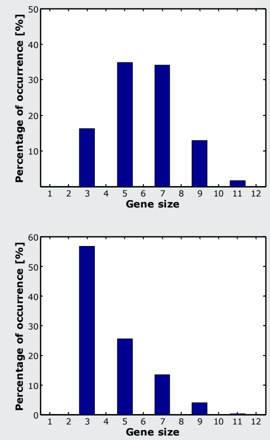
- Gene Expression Programming (GEP)
  - How to remove components that are not needed?





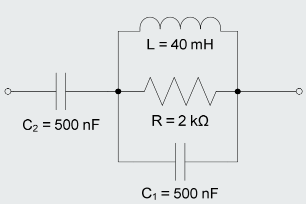
How to identify the impedance circuit?

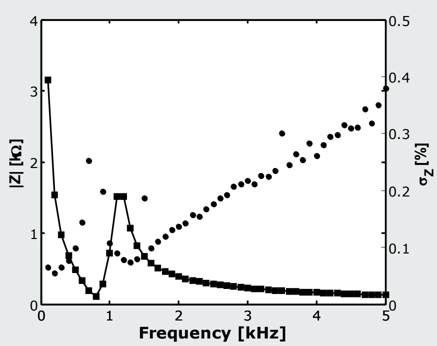
- Gene Expression Programming (GEP)
  - Has this process helped?
    - Average gene size 5.7 → 4.3.
    - Average number of iterations 6.9 → 3.8.
    - Convergence to correct circuit 15.6 % → 56.8 %.
    - Convergence 96 % → 100 %.

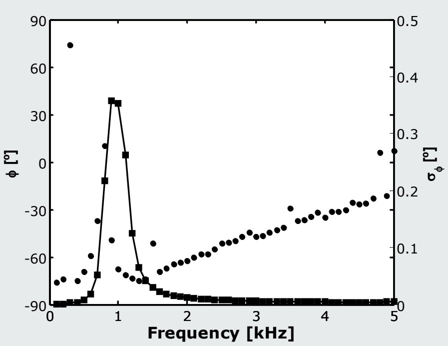


How to identify the impedance circuit?

- Gene Expression Programming (GEP) MEASUREMENT results





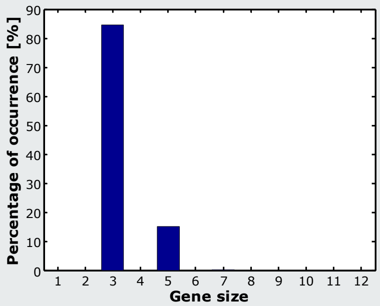


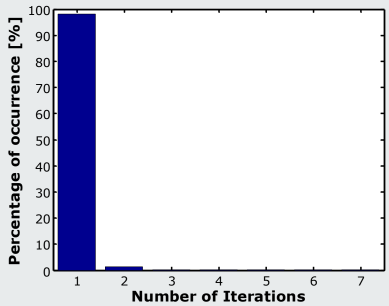
TÉCNICO LISBOA

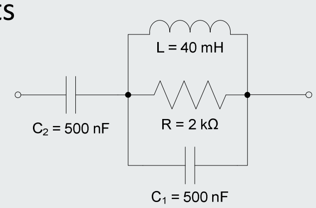
instituto de telecomunicações

How to identify the impedance circuit?

- Gene Expression Programming (GEP) MEASUREMENT results







Experimental standard deviation:

R: 0.11 %

C_1: 0.06 %

L: 0.06 %

C_2: 0.05 %

TÉCNICO LISBOA

instituto de telecomunicações

How to identify the impedance circuit?

- Vector Fitting (VF)
  - The VF algorithm estimates the coefficients of two polynomials whose ratio fits the measured IFR.
  - The algorithm does not estimate the circuit topology.
  - However, from the polynomials an iterative procedure to retrieve the circuit topology has been developed.
  - This is still a work in progress….

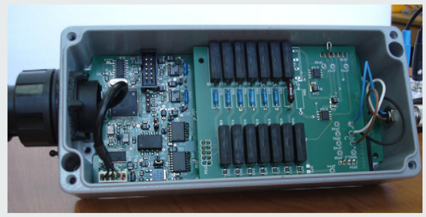
TÉCNICO LISBOA

instituto de telecomunicações

GIm Instrumentation and Measurement Group

Examples of embedded impedance measurement systems

- A Low Cost Miniaturized Impedance Analyzer (2010)
  - Based on two AD5933, 1 MHz, 12-bit impedance converter, network analyzer from Analog Devices.
  - Impedance range: 10 Ω up to 10 GΩ.
  - Frequencies: 0.01 Hz up to 100 kHz.
  - Maximum errors of 2 % and 2.5 °.



J. Hoja and G. Lentka,
"Interface circuit for impedance sensors using two specialized single-chip microsystems",
Sensors and Actuators A: Physical, vol. 163, pp. 191-197, 2010.
doi: 10.1016/j.sna.2010.08.002

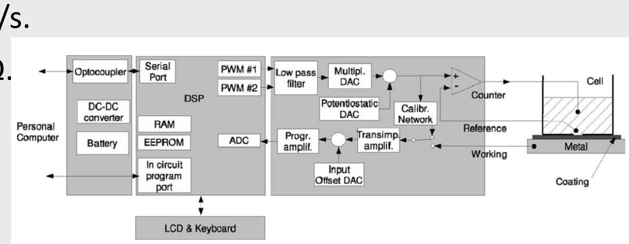
TÉCNICO LISBOA

instituto de telecomunicações

GIm Instrumentation and Measurement Group

Examples of embedded impedance measurement systems

- Seven-Decade Handheld-Impedance-Measurement System (2015)
  - Specifically designed for electrochemical measurements.
  - Processor MSP430 with 12-bit, 200 kS/s.
  - Impedance range: 100 Ω up to 100 GΩ.
  - Frequencies: 0.01 Hz up to 100 kHz.
  - Uncertainty of 5 % and 3 °.



E. Angelini, A. Carullo, S. Corbellini, F. Ferraris, V. Gallone, S. Grassini, M. Parvis and A. Vallan,
"Handheld-impedance-measurement system with seven-decade capability and potentiostatic function",
IEEE Transactions on Instrumentation and Measurement, vol. 55, no. 2, pp. 436-441, Apr. 2006,
doi: 10.1109/TIM.2006.870103

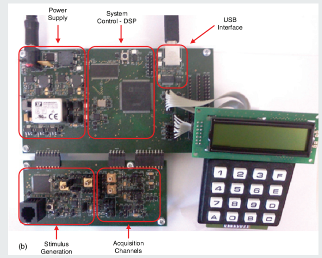
TÉCNICO LISBOA

instituto de telecomunicações

GIM Instrumentation and Measurement Group

Examples of embedded impedance measurement systems

- Embedded System for Viscosity Measurements (2015)
  – Based on a Analog Devices DSP with two external 16-bit ADCs with 1 MS/s.
  – Stimulus from a 14-bit DAC.
  – Frequencies: 100 Hz up to 10 kHz.
  – Multi-harmonic stimulus with multi-harmonic fit.
  – 4-wire connection to impedance.
  – Sensor has very low impedance (near 1 Ω).



J. Santos, F. Janeiro and P. M. Ramos,
"Development, implementation, and characterization of a standalone embedded viscosity measurement system based on the impedance spectroscopy of a vibrating wire sensor", Measurement Science and Technology, vol. 26, n.º 10, pp. 105903-1-14, Oct. 2015.
doi: 10.1088/0957-0233/26/10/105903

TÉCNICO LISBOA

instituto de telecomunicações

GIM Instrumentation and Measurement Group

Conclusions

- Very old topic!
- Major recent developments within the field due to evolution of analog electronics, ADCs and portable low cost digital signal processing.
- Application in many fields.

The end

- Thank you for your attention

Pedro M. Ramos
pedro.m.ramos@tecnico.ulisboa.pt

Google Scholar list of publications