Far side of the ADC: How to Correctly Cope with the Digitization

A discussion on the ADC Dynamic Range

International Ph.D. Excellence School

ITALO GORINI 2021

Ph.D. School ITALO GORINI

Context

- Internet of Things (IoT): processing of multiple signals with a wide range of power at the receiver at lower power consumption [1][2].
- The sensibility of the receiver strongly depends on its dynamic range (DR) which is theoretically directly linked to the ADC resolution [3].

$$(DR)_{dB} = 20\log_{10}\left(\frac{\text{System Full Scale}}{\text{Minimum Value}}\right)$$

- DR have been studied a lot in Compressive Sensing [4], [5] and Communications [6].

ADC requirements already studied for a single signal [7, 8, 9], **BUT**

How to size the **ADC resolution** placed just after the analog front end of digital receivers in oder to **process more than one single signal with high power ratio between them**?

Outline

Minimum resolution for a single tone signal reception

- $x(t)$ be a sine wave of amplitude A
- Quantization step:

$$q = \frac{FSR}{2^{res}}$$

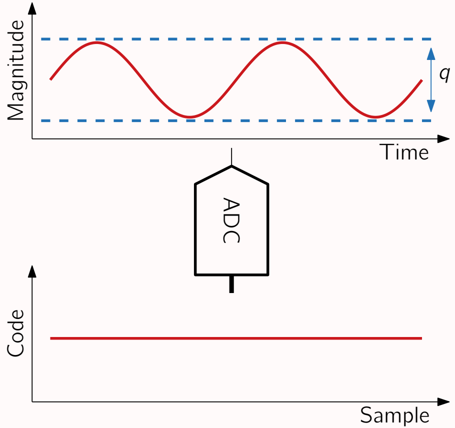
- Signal to noise ratio:

$$(\mathrm{SNR})_{\mathrm{dB}} = 6.02 res + 1.76 + 20 \log_{10}\left(\frac{2A}{FSR}\right)$$

- Dynamic range:

$$\mathrm{DR}|_{\mathrm{dB}} = 20 \log_{10}\left(\frac{FSR}{q}\right) = 6.02 \times res$$

$2A < q$



Minimum resolution for a single tone signal reception

- $x(t)$ be a sine wave of amplitude A
- Quantization step:

$$q = \frac{FSR}{2^{res}}$$

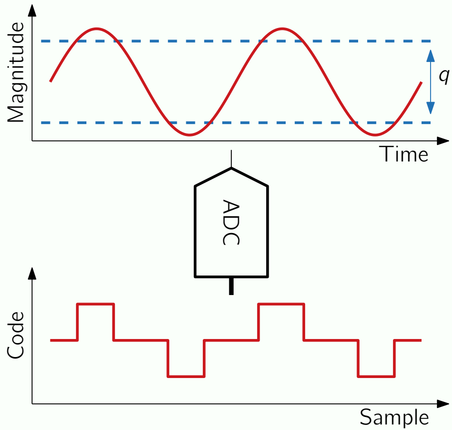
- Signal to noise ratio:

$$(\mathrm{SNR})_{\mathrm{dB}} = 6.02 res + 1.76 + 20 \log_{10}\left(\frac{2A}{FSR}\right)$$

- ADC resolution requirement:

$$res \geq \frac{(\mathrm{DR}_{input})_{\mathrm{dB}}}{6.02}$$

$2A > q$



Two simultaneous analog signals - Vallerian approach [10]

- Vallerian & al. [10] define the dynamic range as:

$$DR_{ws} = 2^{res_{ws}} q = \frac{(x_w + x_s)\sqrt{2}}{K}$$

  - x_s and x_w are respectively the RMS value of the strong and weak signal.
  - K is a scaling factor introduced when a signal is not ranging the full scale.

- The weak signal to quantization noise ratio (SNR_w) can be obtained as:

$$SNR_w = \frac{x_w}{\varepsilon_{RMS}} = K \frac{x_w}{x_w + x_s} 2^{res_{ws}+1} \sqrt{\frac{3}{2}}$$

- Considering the SNR value when a single weak signal is received in order to find the necessary ADC resolution to digitize simultaneously a weak and strong signal leads to:

$$res_{ws} = res_w + \log_2\left(1 + \frac{x_s}{x_w}\right) - \log_2(K)$$

with $res_w = 5$ the resolution for a single weak signal in [10]

Two simultaneous analog signals - Vallerian approach [10]

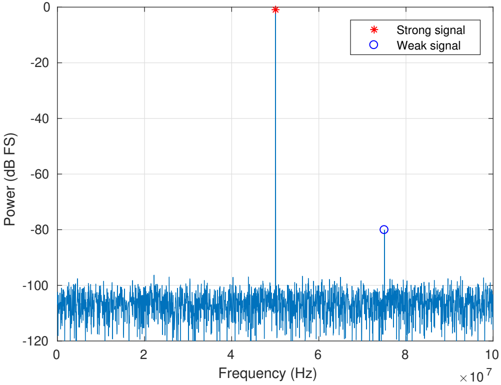
Drawbacks

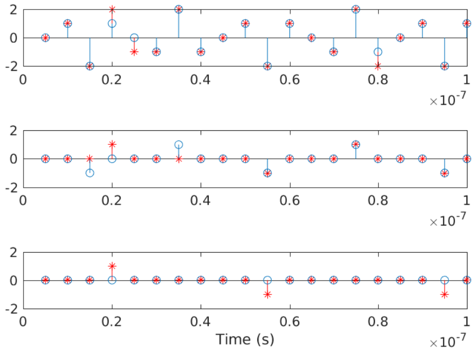
- 21 bits are required to achieve a 100 dB dynamic range depending on the value of K.
- It is also troubling the resolution res_{ws} is deteriorated compared to the one in [3] for a single weak signal reception.
- In fact, some primary results (developed in the next section) shows the opposite with an enhance dynamic range in that particular case.
- These observed minimum ADC resolutions for the correct reception of two signals are overvalued.

Objectifs

- To discuss a new resolution requirement for a given power ratio which is defined as the ratio between the full-scale and the weak signal.
- In this way, the dynamic range can be seen as the maximum poser ratio achievable without the loss of the weak signal.

Observations of strong and weak signals - 12-bits ADC resolution





- The weak signal is still visible even with a power ratio greater than $(DR)_{dB} = 72$dB.
- A signal with $(PR)_{dB} = 20\log_{10}(FSR/A_i) = 72$ dB alone (o) cannot be seen while it can be received when carried by a strong signal $(*)$

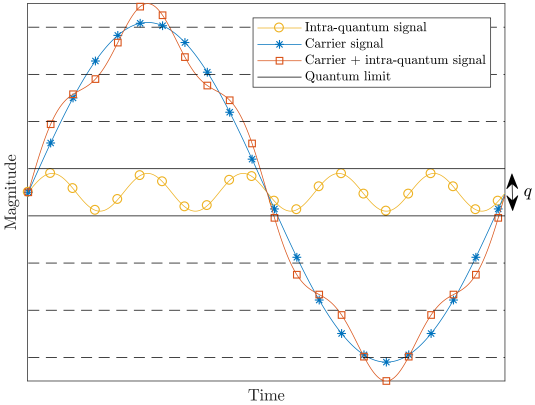
Quantum limitations and signal carriyng

- The ADC is fed by $y(t)$

$$y(t) = x_i(t) + x_c(t)$$

where

- $x_i(t) = A_i \sin(2\pi f_i t + \phi_i)$ (intra-quantum)
- $x_c(t) = A_c \sin(2\pi f_c t + \phi_c)$ (carrier)



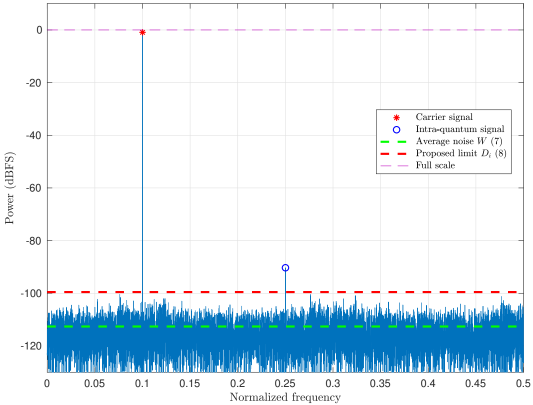
Quantum limitations and signal carriyng

- The ADC is fed by $y(t)$

$$y(t) = x_i(t) + x_c(t)$$

where

- $x_i(t) = A_i \sin(2\pi f_i t + \phi_i)$ (intra-quantum)
- $x_c(t) = A_c \sin(2\pi f_c t + \phi_c)$ (carrier)



Even if the intra-quantum is digitized, a new limit appears: the quantization noise floor introduced by the digitization of $y(t)$ in its spectral representation

New metric to detect an intra-quatum signal

- This metric is estbalish considering the power spectral density of the quantization noise:

$$y(nT_s) = x_i(nT_s) + x_c(nT_s) + n_q(nT_s)$$

with T_s the sampling time and $n_q(nT_s) \sim \mathcal{N}_{\mathbb{R}}(0, \sigma_q^2 = \frac{q^2}{12})$ the quantization noise [11][12].

- The noise floor is :

$$W|_{\text{dBFS}} = -6.02 \times res - 10\log_{10}(N - 3) + 1.24$$

- The variance of the periodogram of a complex white noise process $w[n]$ with variance σ_w^2:

$$\text{Var}[\widehat{P}_w(e^{jw})] = \sigma_w^4 \quad [13][14]$$

$$D_i|_{\text{dBFS}} = W|_{\text{dBFS}} - 10\log_{10}\left(\frac{\left(\frac{q^2}{12N}\right)^2}{W^2}\right) \quad [15]$$

New metric to detect an intra-quatum signal

- This metric is estbalish considering the power spectral density of the quantization noise:

$$y(nT_s) = x_i(nT_s) + x_c(nT_s) + n_q(nT_s)$$

with T_s the sampling time and $n_q(nT_s) \sim \mathcal{N}_{\mathbb{R}}(0, \sigma_q^2 = \frac{q^2}{12})$ the quantization noise [11][12].

- The noise floor is :

$$W|_{\text{dBFS}} = -6.02 \times res - 10\log_{10}(N-3) + 1.24$$

- The variance of the periodogram of a complex white noise process $w[n]$ with variance σ_w^2:

$$\text{Var}[\widehat{P}_w(e^{jw})] = \sigma_w^4 \quad [13][14]$$

- Signal power requirement according to the ADC resolution and for $N = 4096$

res (bits)	**6**	**8**	**10**	**12**	**14**	**16**
$D_i\|_{\text{dBFS}}$	-40	-59	-77	-95	-113	-130

New metric to detect an intra-quatum signal

- This metric is estbalish considering the power spectral density of the quantization noise:

$$y(nT_s) = x_i(nT_s) + x_c(nT_s) + n_q(nT_s)$$

with T_s the sampling time and $n_q(nT_s) \sim \mathcal{N}_{\mathbb{R}}(0, \sigma_q^2 = \frac{q^2}{12})$ the quantization noise [11][12].

- The noise floor is :

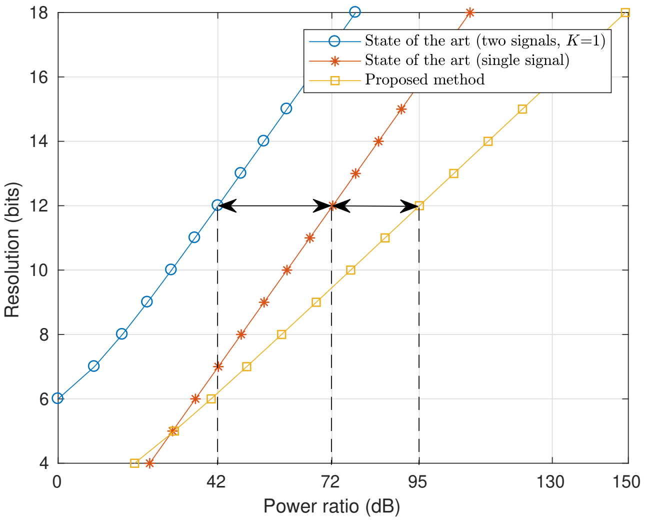
$$W|_{\text{dBFS}} = -6.02 \times res - 10\log_{10}(N-3) + 1.24$$

- The variance of the periodogram of a complex white noise process $w[n]$ with variance σ_w^2:

$$\text{Var}[\widehat{P}_w(e^{jw})] = \sigma_w^4 \quad [13][14]$$

- Limitation: it is based on the detection of the intra-quantum signal through a FFT representation
- Advantage: it allows us to have an idea of the minimum magnitude that can be detected

Comparison between ADC resolution sizing methods



Outline

New context, new metric and some definitions

- The complex envelope of the sampled transmitted signal is:

$$s_I(nT_s) = \sum_{k=0}^{K-1} a_k g(nT_s - kT)$$

  - $a_k = I_k + jQ_k$ with $I_k, Q_k \in \mathbb{R}$
  - T represents the symbol time with $T >> T_s$
  - $g(t)$ is the baseband square-root Nyquist shaping filter
- The transmitted intra-quantum signal is:

$$x_i(t) = A_i s_{I_i}(t) \exp(2\pi f_i t + \phi_i)$$

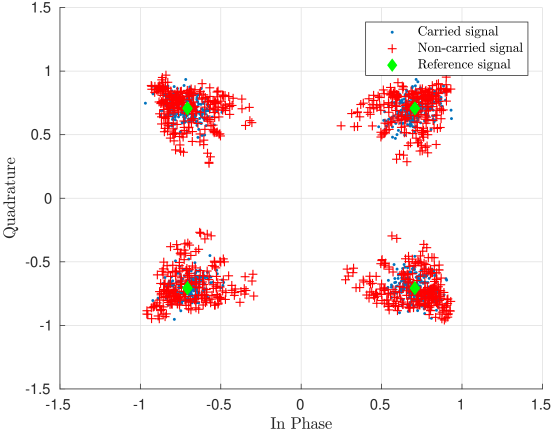
- The transmitted carrier signals is:

$$x_c(t) = A_c s_{I_c}(t) \exp(2\pi f_c t + \phi_c)$$

- Power ratio between the ADC full-scale and the amplitude of the weak signal

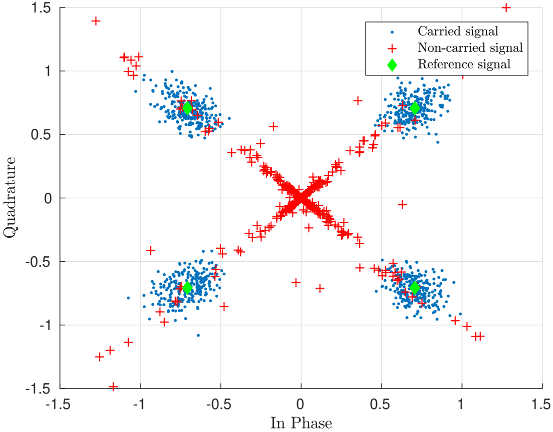
$$(PR)_{dB} = 20 \log_{10}\left(\frac{FSR}{A_w}\right)$$

Simulations results with constellation representation



Constellation comparison for QPSK modulation with a 12-bit ADC, $F_s \times 2$ and a 72 dB power ratio

Simulations results with constellation representation



Constellation comparison for QPSK modulation with a 12-bit ADC, $F_s \times 2$ and a 75 dB power ratio

EVM evaluation

- EVM (Error Vector Magnitude) definition:

$$EVM_{RMS}(\%) = 100 \times \sqrt{\frac{\frac{1}{K}\sum_{k=1}^{K} e_k}{P_{avg}}}$$

where $e_k = \left(I_k - \widetilde{I_k}\right)^2 + \left(Q_k - \widetilde{Q_k}\right)^2$

  - I_k and Q_k represent resp. reference in-phase and quadrature values
  - $\widetilde{I_k}$ and $\widetilde{Q_k}$ represent in-phase and quadrature received symbols.
  - P_{avg} is defined as the average expected constellation power
  - K the vector length.

Power ratio	**72 *dB***	**75 *dB***
EVM carried signal	12 %	20 %
EVM non-carried signal	14 %	96 %

BER results with quantization noise

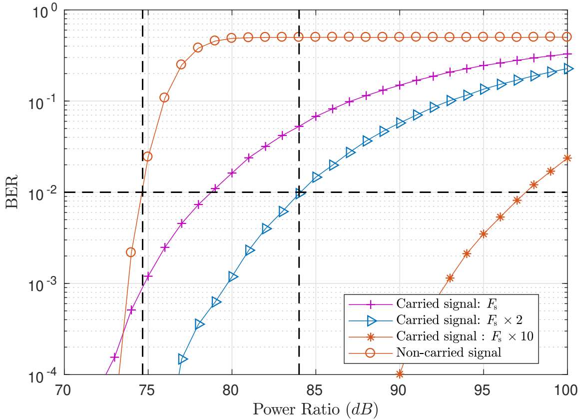
- In this first case, we only consider the quantization noise.
- BER is commonly evaluate as function of the signal to noise ratio. In our case we consider the PR where only the weak signal varies.
- The processing gain, $10\log_{10}(\frac{N}{2})$ for a spectrum analyzer, can also be expressed by using the sampling frequency F_s and the signal bandwidth B [16]:

$$10\log_{10}\left(\frac{F_s}{2B}\right)$$

- It leads to a new definition of the Dynamic Range limit:

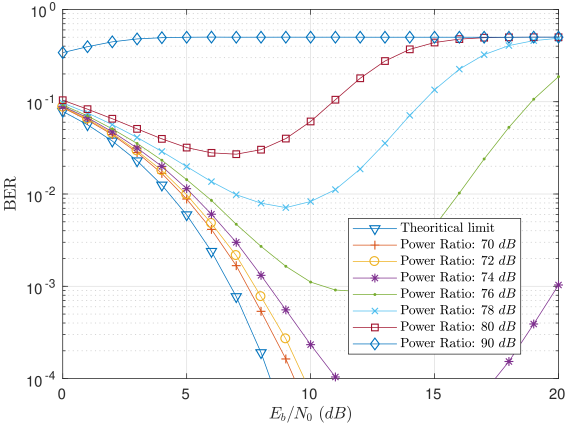
$$D_i|_{\mathrm{dBFS}} = W|_{\mathrm{dBFS}} - 10\log_{10}\left(\frac{\left(\frac{q^2 B}{12F_s}\right)^2}{W^2}\right)$$

BER results with quantization noise



BER comparison between the weak non-carried signal and the weak carried by a stronger one for QPSK modulation with a 12-bit ADC and for different sampling frequencies

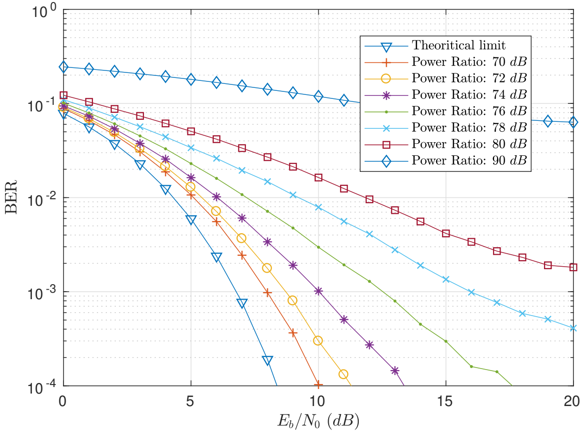
BER results with thermal noise



$$P_b = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

BER comparison for QPSK modulation with a 12-bit ADC, $F_s \times 2$ and different power ratios: single signal reception

BER results with thermal noise

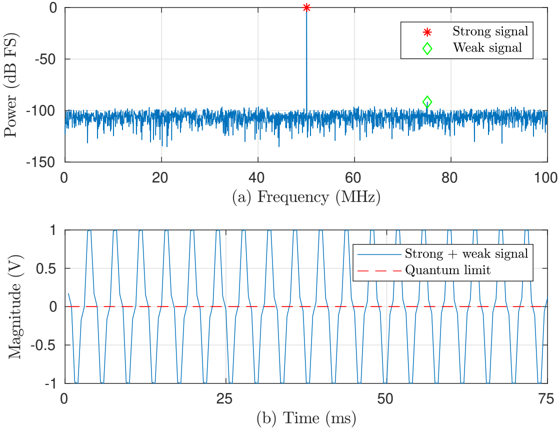


$$P_b = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

BER comparison for QPSK modulation with a 12-bit ADC, $F_s \times 2$ and different power ratios: carried signal reception

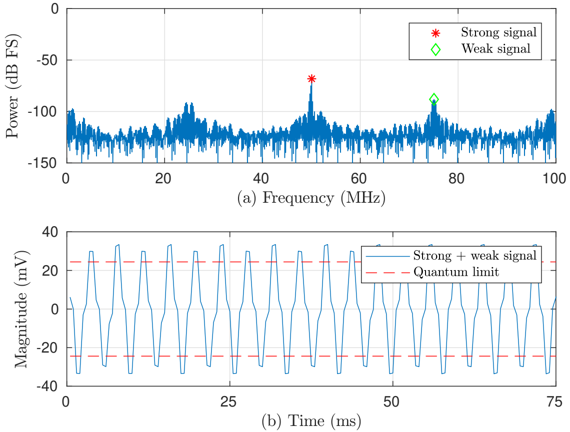
Outline

Strong Signal power limit - Observations



Spectral (a) and time-domain (b) representation with $N = 4096$ FFT points of a dual tone signal with a weak signal at -92 dBFS, the strong signal at 0 dBFS and 12-bit ADC resolution

Strong Signal power limit - Observations



Spectral (a) and time-domain (b) representation with $N = 4096$ FFT points of a dual tone signal with a weak signal at -92 dBFS, the strong signal at -70 dBFS and 12-bit ADC resolution

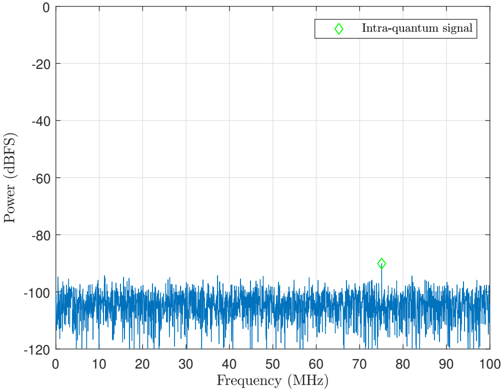
Inherent noise as carrier signal

- Electronic noise like the thermal noise.
- Before the quantification, the sampled signal is:

$$y(nT_s) = x_i(nT_s) + n_l(nT_s)$$

with $n_l(nT_s) \sim \mathcal{N}_{\mathbb{R}}(0, \sigma_n^2)$

- Spectral representation with $N = 4096$ FFT points, intra-quantum signal at -92 dBFS, SNR = -70 dBFS and $res = 12$-bit



The position of the noise is coherent with the chosen SNR considering the FFT processing gain

Comparison of carrier signals

What is the percentage of intra-quantum signal detection as function of the carrier signal: $n_l(t)$ or $x_c(t)$?

Percentage of detection for different carrier signals and different power ratios

$$PR|_{dB} = 20\log_{10}\left(\frac{FSR}{2A_i}\right)$$

PR (dB)	70	75	80	85	90	95	100
$x_c(t)$ (-1 dBFS)	100	100	100	100	96.4	7.5	1.3
$x_c(t)$ (-10 dBFS)	100	100	100	100	94	4.6	0
$x_c(t)$ (-30 dBFS)	100	93.9	91.1	9.3	0	0	0
$n_l(t)$ (-70 dBFS)	100	99.9	99.7	98.1	9.7	0.2	0.2

A 12-bit ADC and $N = 4096$ was considered. It means that, for the 70 dB column, the weak signal is not intra-quantum since $70 < 72$ dB.

Comparison of carrier signals

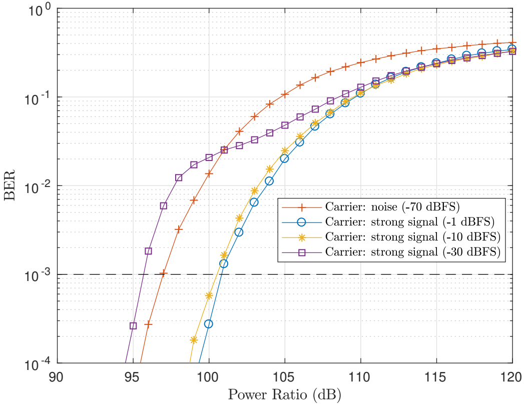
- Strong signal chosen as carrier:

$$y(nT_s) = x_i(nT_s) + x_c(nT_s)$$

- Intrinsic noise chosen as carrier:

$$y(nT_s) = x_i(nT_s) + n_I(nT_s)$$

- SNR = -70 dBFS
- QPSK modulation
- $F_s = 100$ MHz and $B = 25$-kHz

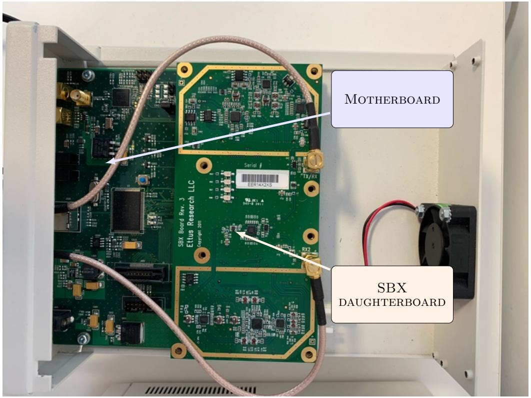


First conclusions

1. The strong signal appears to be more interesting than the inherent noise since it contributes only on a narrow band or, in the case of single tone signals, on a single frequency.
2. The strong signal does not need to be close to the full-scale to carry an intra-quantum signal even if we can observe some degradation when it becomes too weak. This result can be explained by the quantization noise which loses its white behavior and makes the detection by thresholding more difficult.

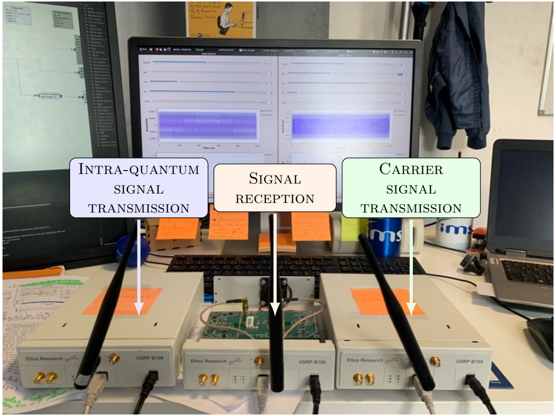
Outline

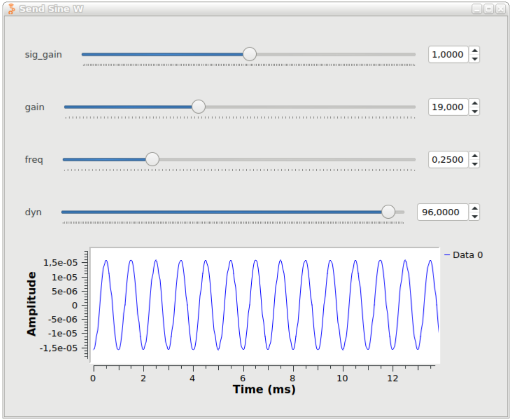
USRP B100 Software Defined Radio



Model	ADC	DAC	Max Gain (dB)	RF Carrier frequencies (MHz)
B100	12-bit (64 MS/s)	14-bit (128 MS/s)	31.5	400 to 4400

USRP B100 Software Defined Radio

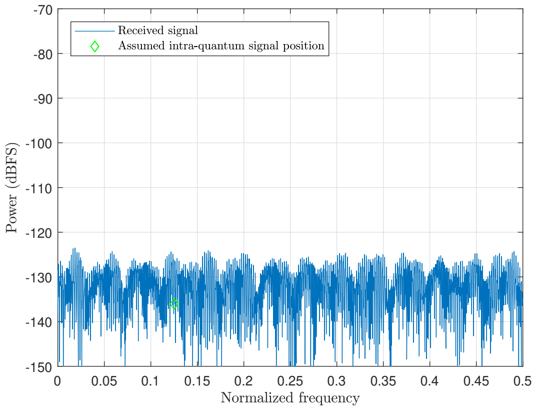




Weak single tone signal alone

The intra-quantum signal is adjusted so that the signal disappear on the receiver FFT

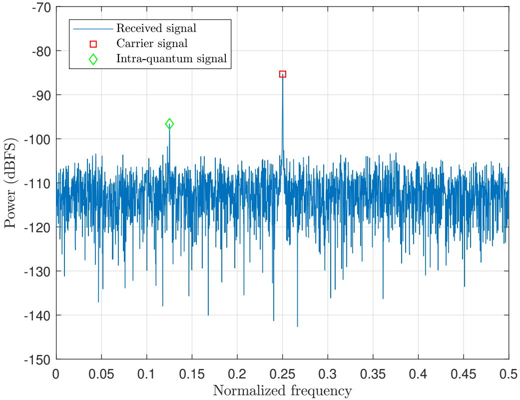
- Observation of the theoretical limitation of a 12-bit ADC receiver.
- The quantum limit is still not the theoretical one, due to the thermal noise will also make the intra-quantum signal cross the quantum level.



Intra-quantum and carrier single tone signals

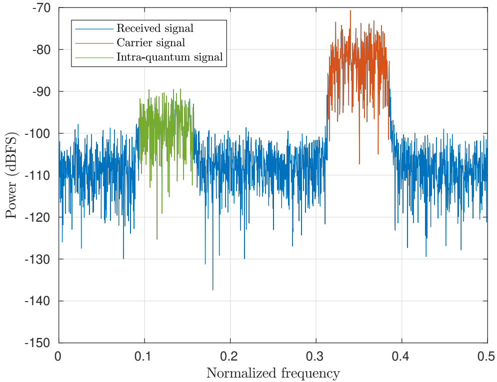
The strongest signal did not need to be necessarily close to the full scale in order to make the weakest one cross the quantum level

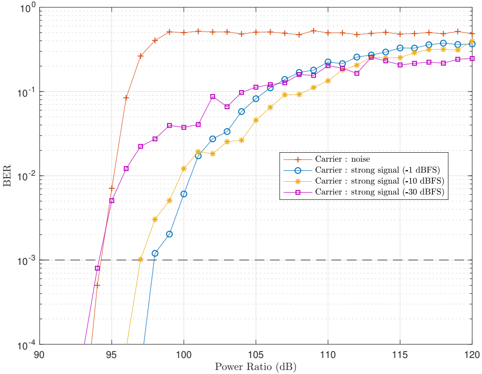
- Strong signal is generated just strong enough to make it cross the quantum levels.
- Experimental results are coherent with the simulation limit which underlined a −95 dBFS.
- Below this value, the intra-quantum signal is still carried but meddled with the noise floor and thus irretrievable without more advanced signal processing techniques.



Digital communication case with a QPSK modulated signal

QPSK modulation with $Fs = 64$ MHz and $B = 15$ kHz

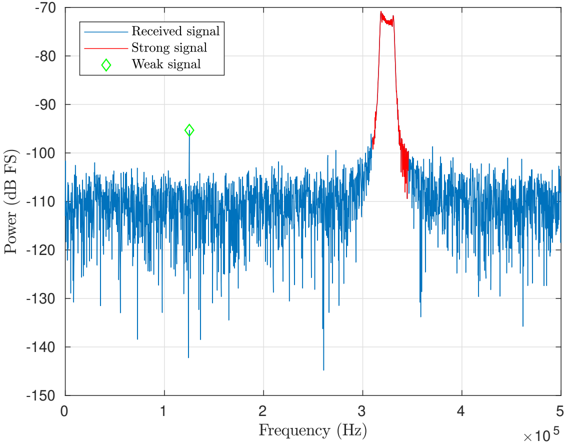




Weak single tone signal and IoT LoRa signal

LoRa node transmitting on a 443 MHz carrier frequency

Weak single tone signal and IoT LoRa signal



Weak signal received with a stronger IoT LoRa signal

Outline

Conclusion

- A new way to size an ADC by its resolution is proposed.
- The theoretical dynamic range was no longer relevant and needs to be adapted to nowadays sampled bandwidths which include more and more signals with high power ratios.
- We proposed a metric based on a frequency analysis and compared the influence of the carrier waveform.
- Finally, these results were confronted with real transmission by using SDR hardware [17, 18].
- This work could be extended to the consideration of other ADC architectures such as Sigma-Delta ADC where the quantization noise can be prevalent
- However, even though this work underlines the overvalued ADC requirements, it will not substitute the need of new technics to overtake the bounded ADC dynamic range such as companding techniques [19, 20].

For Further Reading I

[1] L. Mainetti, L. Patrono, and A. Vilei.
Evolution of wireless sensor networks towards the internet of things: A survey.
In *SoftCOM 2011, 19th International Conference on Software, Telecommunications and Computer Networks*, pages 1–6, 2011.

[2] Guillaume Ferré and Eric Pierre Simon.
Packet collision analysis when heterogeneous unlicensed IoT technologies coexist.
IET Networks, 7(6):384–392, nov 2018.

[3] D. Van Ess.
Algorithm - logarithmic signal companding - not just a good idea - it is μ-law.
Cypress Application Notes, AN2095, December 2013.

For Further Reading II

[4] Mark A. Davenport, Jason N. Laska, John R. Treichler, and Richard G. Baraniuk.
The pros and cons of compressive sensing for wideband signal acquisition: Noise folding versus dynamic range.
IEEE Transactions on Signal Processing, 60(9):4628–4642, September 2012.

[5] J. Treichler, M. Davenport, Jason N. Laska, and Richard Baraniuk.
Dynamic range and compressive sensing acquisition systems.
In *Defense Applications of Signal Processing (DASP)*, pages 1–7, 2011.

[6] Budhaditya Banerjee, Christian C. Enz, and Erwan Le Roux.
Detailed analysis of a phase ADC.
In *Proceedings of 2010 IEEE International Symposium on Circuits and Systems*. IEEE, May 2010.

For Further Reading III

[7] Ieee standard for terminology and test methods for analog-to-digital converters.
IEEE Std 1241-2010 (Revision of IEEE Std 1241-2000), pages 1–139, 2011.

[8] Shenghua Zheng, Dazhuan Xu, and Xueming Jin.
ADC limitations on the dynamic range of a digital receiver.
In *2005 IEEE International Symposium on Microwave, Antenna, Propagation and EMC Technologies for Wireless Communications*, volume 1, pages 79–82, 2005.

[9] Bengt E. Jonsson.
A survey of a/d-converter performance evolution.
In *2010 17th IEEE International Conference on Electronics, Circuits and Systems*, pages 766–769, December 2010.

For Further Reading IV

[10] Mathieu Vallerian, Guillaume Villemaud, Benoit Miscopein, Tanguy Risset, and Florin Hutu.
SDR for SRD: ADC specifications for reconfigurable gateways in urban sensor networks.
In *2014 IEEE Radio and Wireless Symposium (RWS)*, pages 178–180, January 2014.

[11] A. Sripad and D. Snyder.
A necessary and sufficient condition for quantization errors to be uniform and white.
IEEE Transactions on Acoustics, Speech, and Signal Processing, 25(5):442–448, October 1977.

[12] B. Widrow, I. Kollar, and Ming-Chang Liu.
Statistical theory of quantization.
IEEE Transactions on Instrumentation and Measurement, 45(2):353–361, April 1996.

For Further Reading V

[13] Charles W. Therrien.
Discrete random signals and statistical signal processing.
Prentice Hall PTR, 1992.

[14] Petre Stoica and Randolph L. Moses.
Introduction to Spectral Analysis.
Prentice Hall PTR, 1997.

[15] Baptiste Laporte-Fauret, Guillaume Ferre, Dominique Dallet, Bryce Minger, and Loic Fuche.
ADC resolution for simultaneous reception of two signals with high dynamic range.
In *2018 25th IEEE International Conference on Electronics, Circuits and Systems (ICECS)*, pages 729–732, December 2018.

For Further Reading VI

[16] Baptiste Laporte-Fauret, Guillaume Ferre, Dominique Dallet, Bryce Minger, and Loic Fuche. ADC required resolution for correct demodulation of an intra-quantum signal. In *2019 IEEE International Symposium on Circuits and Systems (ISCAS)*, pages 1–5, May 2019.

[17] Baptiste Laporte-Fauret, Guillaume Ferre, Dominique Dallet, Bryce Minger, and Loic Fuche. Intra-quantum signal acquisition using software defined radios. In *2019 IEEE International Instrumentation and Measurement Technology Conference (I2MTC)*, pages 1–6, May 2019.

[18] Baptiste Laporte-Fauret, Guillaume Ferre, Dominique Dallet, Bryce Minger, and loic Fuche. ADC resolution requirement for an intra-quantum signal digitization. *IEEE Transactions on Instrumentation and Measurement*, 2020.

For Further Reading VII

[19] Mathieu Vallerian, Florin Hutu, Benoit Miscopein, Guillaume Villemaud, and Tanguy Risset.
Additive companding implementation to reduce ADC constraints for multiple signals digitization.
In *2015 IEEE 13th International New Circuits and Systems Conference (NEWCAS)*, pages 1–4, June 2015.

[20] J. Bes Cees, Chutham Sawigun, and A. Serdijn Wouter.
An additive instantaneously companding readout system for cochlear implants.
In *2010 Biomedical Circuits and Systems Conference (BioCAS)*, pages 126–129, November 2010.