Quantum (electrical) Metrology and the revision of the International System of units (SI)

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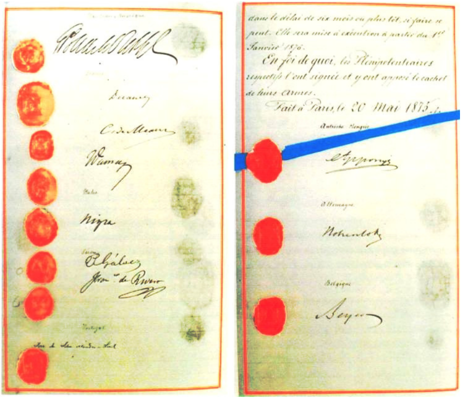
The Metre Convention

Paris, 20 May 1875: an international treaty



dans le délai de six mois ou plus tôt, si faire se peut. Elle sera mise à exécution à partir du 1er Janvier 1876.

En foi de quoi, les Plénipotentiaires respectifs l'ont signée et y ont apposé le cachet de leurs armes.

Fait à Paris, le 20 Mai 1875.

Autriche-Hongrie

Apponyi

Allemagne

Hohenlohe

Belgique

Beyens

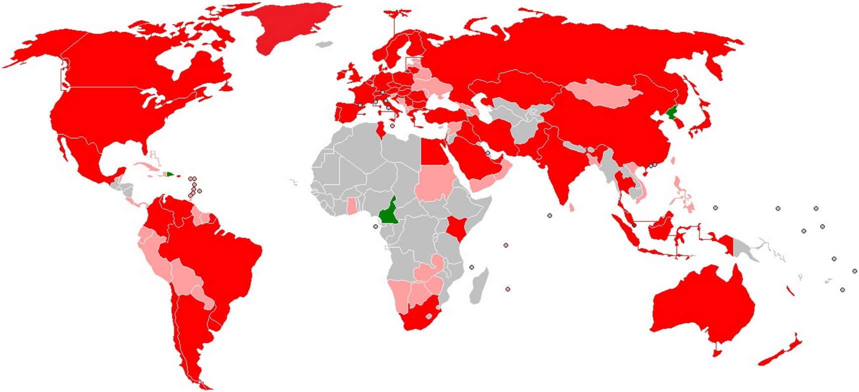
Nigra

Original signatories: Argentina, Austria-Hungary, Belgium, Brazil, Denmark, France, Germany, Italy, Peru, Portugal, Russia, Spain, Sweden and Norway, Switzerland, Turkey, United States of America, and Venezuela

[for His Majesty the King of Italy: Chevalier Constantino Nigra, Knight of the Grand Cross of his Orders of St. Maurice and St. Lazarus, and of the Crown of Italy, Grand Officer of the Legion of Honor, . . . Extraordinary and Minister Plenipotentiary at Paris]

The Metre Convention

The signatories today



The SI, 1960-today : what does *not* change

Base and derived units

Base units

Symbol	Unit name
s	second
m	metre
kg	kilogram
A	ampere
K	kelvin
mol	mole
cd	candela

Base and derived units

Base units

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Derived units

$$\mathrm{s}^{\alpha}\,\mathrm{m}^{\beta}\,\mathrm{kg}^{\gamma}\,\mathrm{A}^{\delta}\,\mathrm{K}^{\epsilon}\,\mathrm{mol}^{\zeta}\,\mathrm{cd}^{\eta},$$

where α, β, γ, δ, ϵ, ζ and η are (usually) integers.

The International System of units (SI

many derived units

Examples

$\mathrm{m/s}$ unit of velocity;

$\mathrm{W = kg\,m/s^2}$ unit of power;

$\mathrm{V = kg m^2 s^{-3} A^{-1}}$ unit of electrical potential difference (voltage)

SI units for electromagnetic quantities

Derived units with special names

Derived quantity	name	symbol	expression in terms of base units
frequency	hertz	Hz	s^{-1}
energy	joule	J	$\mathrm{m}^{2}\,\mathrm{kg}\,\mathrm{s}^{-2}$
power	watt	W	$\mathrm{m}^{2}\,\mathrm{kg}\,\mathrm{s}^{-3}$
electric charge	coulomb	C	$\mathrm{s}\,\mathrm{A}$
electric potential difference	volt	V	$\mathrm{m}^{2}\,\mathrm{kg}\,\mathrm{s}^{-3}\,\mathrm{A}^{-1}$
electric capacitance	farad	F	$\mathrm{m}^{-2}\,\mathrm{kg}^{-1}\,\mathrm{s}^{-4}\,\mathrm{A}^{2}$
electric resistance	ohm	Ω	$\mathrm{m}^{2}\,\mathrm{kg}\,\mathrm{s}^{-3}\,\mathrm{A}^{-2}$
electric conductance	siemens	S	$\mathrm{m}^{-2}\,\mathrm{kg}^{-1}\,\mathrm{s}^{3}\,\mathrm{A}^{2}$
magnetic flux	weber	Wb	$\mathrm{m}^{2}\,\mathrm{kg}\,\mathrm{s}^{-2}\,\mathrm{A}^{-1}$
magnetic flux density	tesla	T	$\mathrm{kg}\,\mathrm{s}^{-2}\,\mathrm{A}^{-1}$
inductance	henry	H	$\mathrm{m}^{2}\,\mathrm{kg}\,\mathrm{s}^{-2}\,\mathrm{A}^{-2}$

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electric resistance	ohm	Ω	$\mathrm{m}^2\ \mathrm{kg}\ \mathrm{s}^{-3}\ \mathrm{A}^{-2}$
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inductance	henry	H	$\mathrm{m}^2\ \mathrm{kg}\ \mathrm{s}^{-2}\ \mathrm{A}^{-2}$

Remark

Can form further derived units. For example, permittivity can be expressed either in F/m or in $\mathrm{m}^{-3}\,\mathrm{kg}^{-1}\,\mathrm{s}^4\,\mathrm{A}^{-2}$.

SI prefixes and suffixes

The SI adopts a series of prefix names and prefix symbols to form the names and symbols of the decimal multiples and submultiples of units, ranging from 10^{24} to 10^{-24}.

name	symbol	factor	name	symbol	factor
yocto	y	10^{-24}	deca	da	10^{1}
zepto	z	10^{-21}	hecto	h	10^{2}
atto	a	10^{-18}	kilo	k	10^{3}
femto	f	10^{-15}	mega	M	10^{6}
pico	p	10^{-12}	giga	G	10^{9}
nano	n	10^{-9}	tera	T	10^{12}
micro	μ	10^{-6}	peta	P	10^{15}
milli	m	10^{-3}	exa	E	10^{18}
centi	c	10^{-2}	zetta	Z	10^{21}
deci	d	10^{-1}	yotta	Y	10^{24}

The expression of the value of electromagnetic quantities benefits of large or small prefixes, more often than in other scientific fields. For example, it is common to speak of fA current, PΩ resistance, or aF capacitance values.

The SI, 1960-2019

SI, 1960-2019

The seven base units

m The metre is the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second.

kg The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.

s The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom.

A The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

K The kelvin, unit of thermodynamic temperature, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

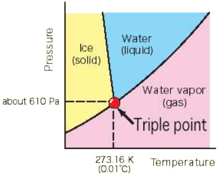
mol The mole is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kg of carbon 12.

cd The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

SI, 1960-2019: Definition of the base units

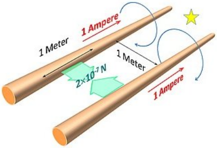
an artefact:

> *The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.*



a natural property

> *The kelvin is the fraction* $1/273.16$ *of the thermodynamic temperature of the triple point of water.*



an idealized experiment

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length [. . .] would produce a force equal to* 2×10^{-7} *newton per metre of length*

SI, 1960-2019: Realization of the units

Realization (VIM 5.1 ↗)

The realization of the definition of a unit can be provided by a measuring system, a material measure, or a reference material.

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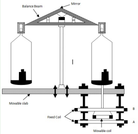
SI 1960-2019:

an artefact:

The international prototype of the kilogram is the realization of the kilogram.

a device

A triple point of water cell is a realization of the kelvin.



an experiment

The current balance is a realization of the ampere.

SI, 1960-2019: the reproduction of the units

Reproduction (VIM 5.1 ↗)

The *reproduction* of a unit consists in realizing the unit not from its definition but in setting up a highly reproducible measurement standard based on a physical phenomenon, and, usually, by assigning to it a conventional value.

SI, 1960-2019: the reproduction of the units

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Examples

In the SI 1960-2019:

- The volt is reproduced by means of the Josephson effect.
- The ohm is reproduced by means of the quantum Hall effect.
- The thermodynamic temperature scale is reproduced through two conventional temperature scales, the *International Temperature Scale of 1990* (ITS-90) and the *Provisional Low Temperature Scale of 2000* (PLTS-2000) through *fixed points* and *interpolators*.

The ampere, 1960-2019

The definition of the base unit ampere is mechanical:

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.*

All electromagnetic derived units have an ultimately mechanical definition also.

These quantities are **exact**:

$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m}$ the *magnetic constant*;

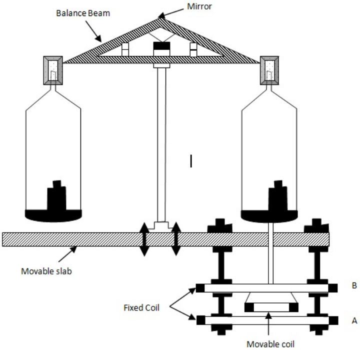
$\epsilon_0 = \left(\mu_0 c^2\right)^{-1} = 8.854\,187\,817\ldots\,\mathrm{pF/m}$, the *electric constant*

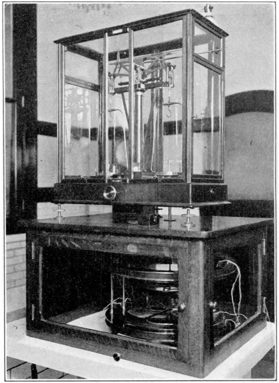
$Z_0 = \mu_0\, c = \sqrt{\mu_0\, \epsilon_0^{-1}} = 376.730\,313\,4\ldots\,\Omega$, the *impedance of free space*

μ_0, ϵ_0 constant $\Rightarrow$ realization of SI units of impedance.

Realization of the ampere

The (electrodynamic) ampere balance (Vigoreux, 1965)





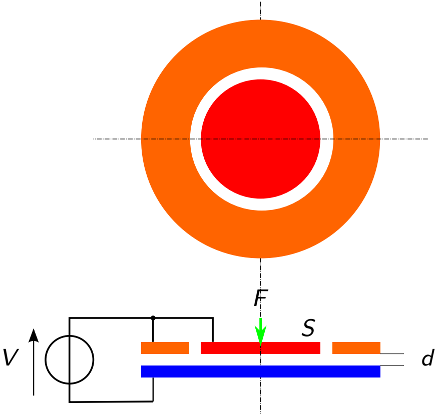
Ampère force law:

$$F = \frac{\mu_0}{4\pi} \int_{\Gamma_1} \int_{\Gamma_2} \frac{I_1 \, \mathrm{d}\ell_1 \times I_2 \, \mathrm{d}\ell_2 \times \boldsymbol{r}_{21}}{|\boldsymbol{r}_{21}|^2}$$

If $I_1 = I_2$, $F = \mu_0 k I^2$ where k is computed from geometrical measurements

Realization of the volt

The (electrostatic) voltage balance



Force between plates: $F = \epsilon_0 \dfrac{S}{2d^2} V^2 = \epsilon_0 \, k \, V^2$

where k is computed from geometrical measurements

Realization of the volt

Cylindrical-electrode voltage balance, PTB (Siencknecht and Funck, 1986)

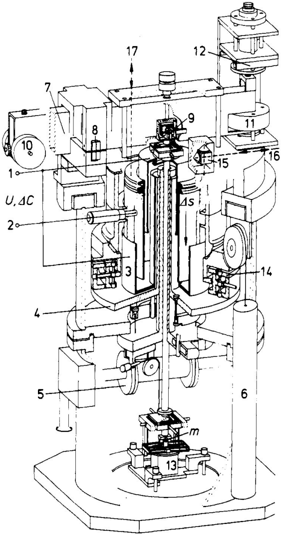
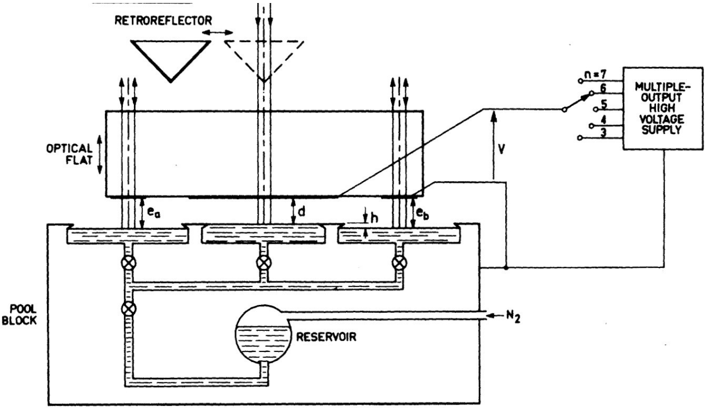


Fig. 1. Perspective view of the PTB voltage balance. 1 Inner electrode, 2 high-voltage electrode, 3 guard electrode, 4 carriage of displace unit, 5 driving device for displace unit, 6 counterweight of displace unit, 7 balance beam, 8 central joint of balance beam, 9 load joint of balance beam, 10 counterbalance weight, 11 position sensor, 12 retainer for balance beam, 13 load-changing device, 14 device for centering and vertical electrode adjustment, 15 interferometer for Δs-measurement, 16 light beam of interferometers for Δs-measurement, 17 light beam of autocollimator for vertical electrode adjustment

$$V = 10\,186\,\mathrm{V} = 1000 \times E_{\mathrm{Weston}};\ m = 2\,\mathrm{g}\ !$$

Realization of the volt

Mercury-electrode elevation, CSIRO Australia (Sloggett et al., 1985)

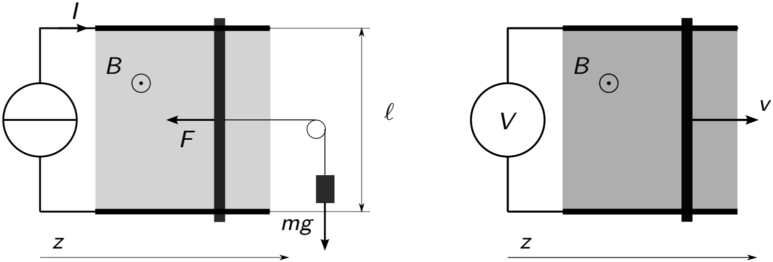


$$V=\sqrt{\frac{2\rho g}{\epsilon_0}}\,d\sqrt{h}.\ \ V=\text{kV},\ d=600\,\mu\text{m},\ u_V=0.33\times10^{-6}$$

Realization of the electrical watt

The watt balance, or Kibble balance

Solves the problem of geometrical measurements!



- Weighing mode: $F = mg = B\ell I = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z} I$
- Moving mode: $E = \dfrac{\mathrm{d}\Phi}{\mathrm{d}t} = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z}\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{\mathrm{d}\Phi}{\mathrm{d}z} v$

$$mgv = EI;$$

mechanical power = electrical power

The Kibble balance

(Robinson and Schlamminger, 2016)

Solves the problem of geometrical measurements!

weighing mode

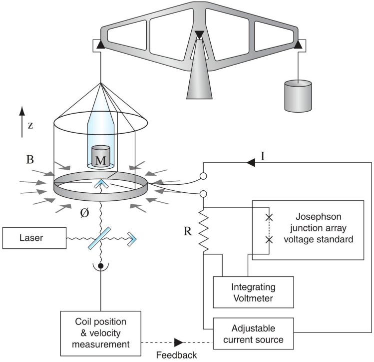


Figure 1. The Kibble balance in weighing mode.

moving mode

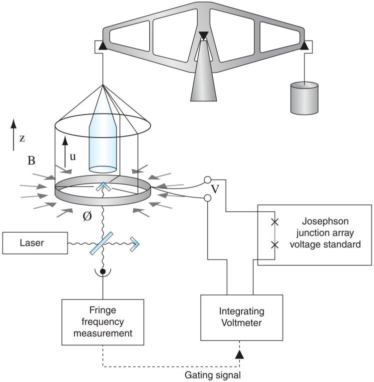
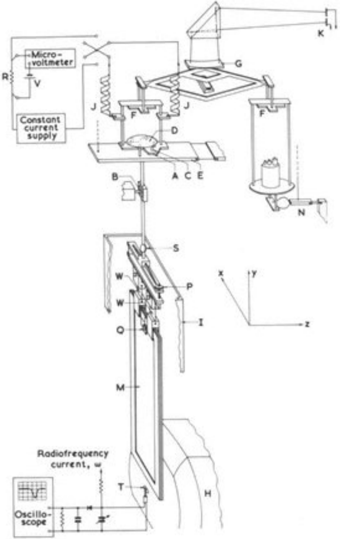


Figure 2. The Kibble balance in moving mode.

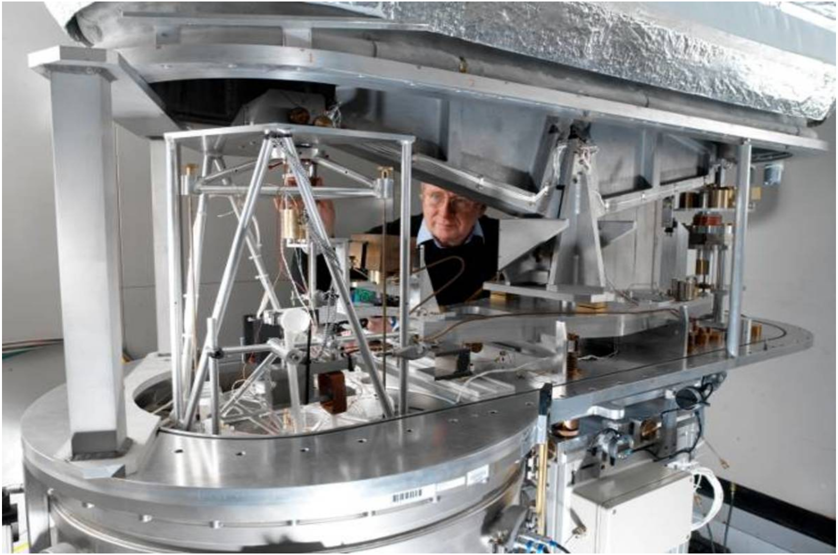
The Kibble balance evolution

NPL, Kibble (1976) for the gyromagnetic ratio of the proton



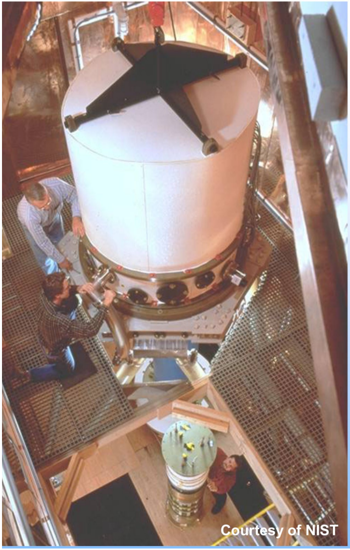
The Kibble balance: evolution

NRC, Bryan P. Kibble and I. Robinson, 2011

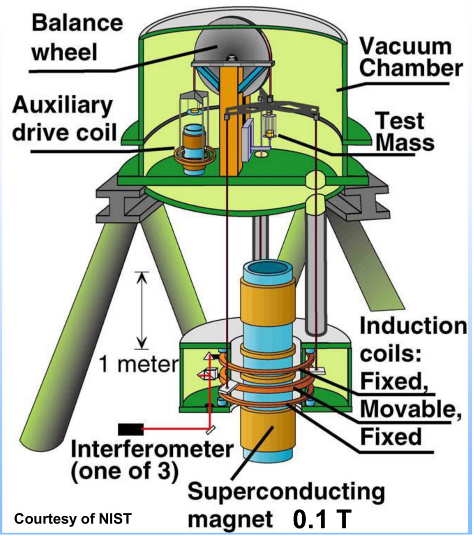


The Kibble balance: evolution

NIST-3



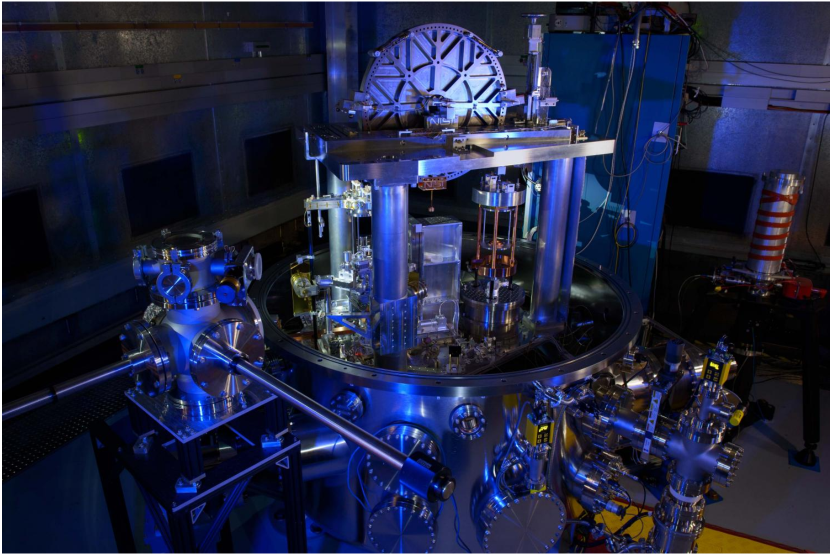
Courtesy of NIST



Courtesy of NIST

The Kibble balance: evolution

The last generation: NIST-4, 2016



The Kibble balance: evolution

The last generation: NPL, 2017

The Kibble balance

Determination of the Planck constant

To be discussed again after the quantum experiments

- $mgv = EI$
- $E = n\frac{f_\mathrm{E}}{K_\mathrm{J}}$
- $I = \frac{V_\mathrm{I}}{R} = \frac{f_\mathrm{I}}{K_\mathrm{J}}\frac{1}{rR_\mathrm{K}}$
- $K_\mathrm{J} = \frac{2e}{h}$
- $R_\mathrm{K} = \frac{h}{e^2}$

$$\Rightarrow mgv = hf_\mathrm{E}f_\mathrm{I}\frac{n}{r}$$

h can be measured mechanically

Realization of impedance units

Calculable geometries

if the geometry of the system of conductors is sufficiently simple, explicit mathematical expressions for their inductance or capacitance value may exist. For example:

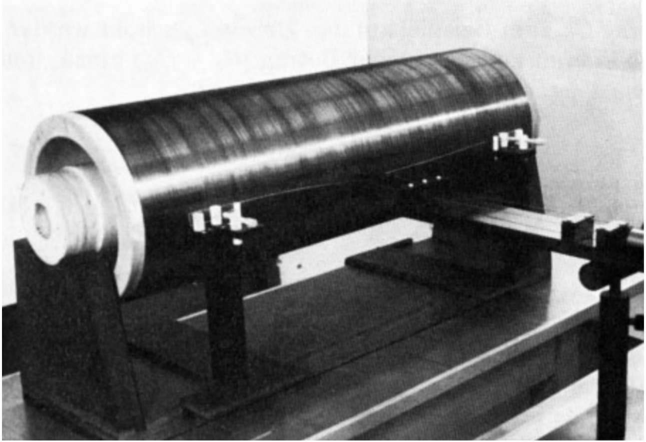
- the low-frequency inductance L of a circular conductive loop (of radius r), made of a circular perfect conductor (of radius a), in vacuum, is $L = \mu_0\, r\, [\log(8r/a) - 7/4]$;
- the capacitance C of a conducting sphere of radius R in vacuum is $C = 4\pi\epsilon_0 R$.

The previous examples are not adequate for a practical impedance realization, which require a careful choice of the calculable geometrical shape of conductors in order to minimize:

- the dependence of L or C on inevitable deviations of the mechanical realization of conductors' shapes from the ideal geometry employed in the mathematical modelling;
- the number, and practical difficulty, of the accurate length measurements which are needed in the calculation.

Realization of impedance units: the henry

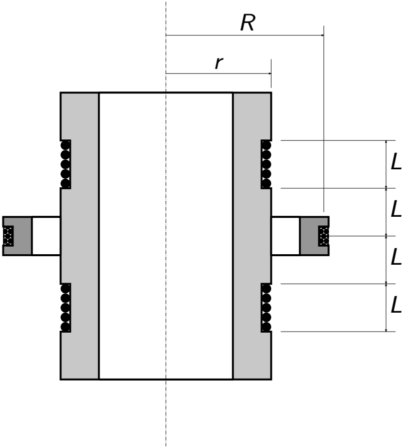
The PTB self-inductor (Linckh and Brasack, 1968)

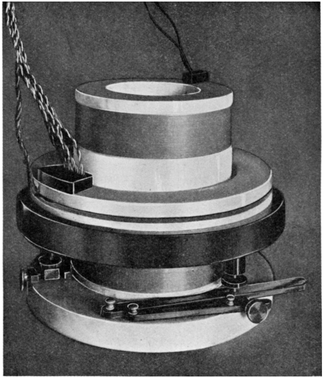


$L = \mu_0 k N^2$ where k is determined by geometrical measurements

Realization of the inductance unit, the henry

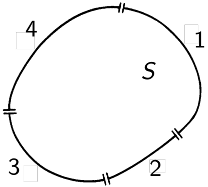
The NPL Mutual inductor (Campbell, 1907)





Realization of the farad

the calculable capacitor



The general geometry of four conductors 1, 2, 3, 4 having cylindrical symmetry, and arranged in a closed shell with infinitesimal gaps, analyzed by the Thompson-Lampard theorem.

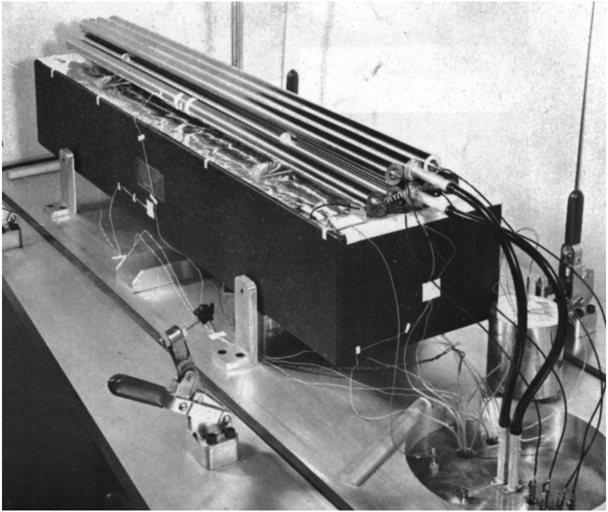
Thompson-Lampard theorem (Lampard, 1957)

$$\exp\left(-\pi\frac{C_{13}}{\epsilon_0}\right) + \exp\left(-\pi\frac{C_{24}}{\epsilon_0}\right) = 1.$$

If there is sufficient symmetry such that $C_{13} = C_{24} = C$,

$$C = \epsilon_0 \frac{\log 2}{\pi} = 1.953549043\ldots \times 10^{-12}\,\mathrm{F/m} \qquad \text{[exact]}.$$

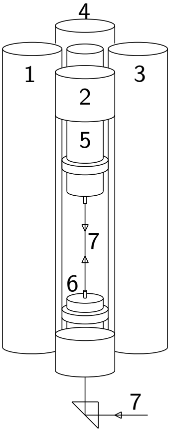
The calculable capacitor



1964: Fixed calculable capacitor, realized with stacked gauge bars, NRC (Dunn, 1964).

Realization of the farad

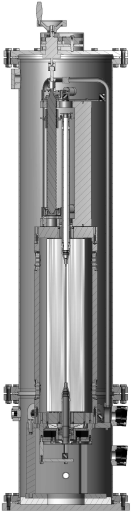
the calculable capacitor



Cross capacitor with movable guard electrode. 1, 2, 3, and 4 are the four cylindrical electrodes to which the cross-capacitor theorem is applies. 5 and 6 are the two guard electrodes; electrode 6 can be moved axially between two positions; the motion is monitored by a laser interferometer 7.

$C = \epsilon_0 \dfrac{\log 2}{\pi} \ell$, where ℓ is a geometrical length to be measured.

The calculable capacitor



2015: NMIA-BIPM cross capacitor, with movable guard. (courtesy of J. Fiander)

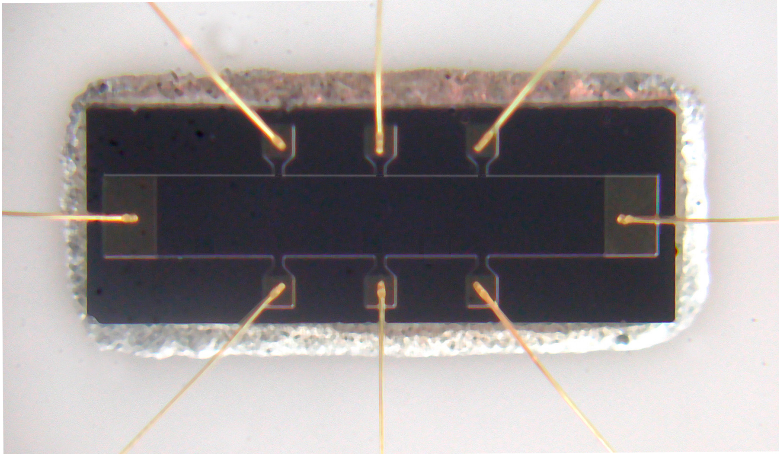
Quantum electrical metrology

Quantum electrical metrology experiments

Macroscopic quantum effect that display an electrical quantity related to fundamental constants

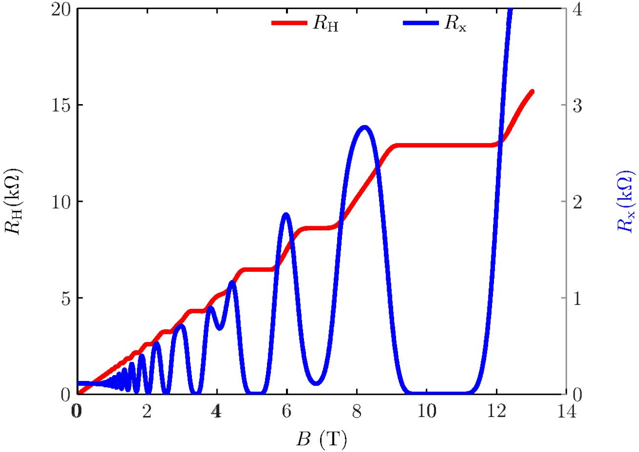
- quantized resistance: the quantum Hall effect
- quantized flux counting: the Josephson effect
- quantized charge counting: single-electron counting devices

The quantum Hall effect



AlGaAs/GaAs Hall bar heterostructure, 1 mm × 0.4 mm;

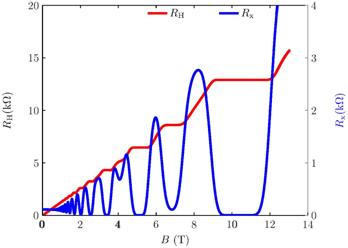
The quantum Hall effect



— $R_{\mathrm{H}} = V_{\mathrm{H}}/I$ Hall resistance;

— $R_{\mathrm{x}} = V_{\mathrm{x}}/I$ longitudinal resistance.

The quantum Hall effect

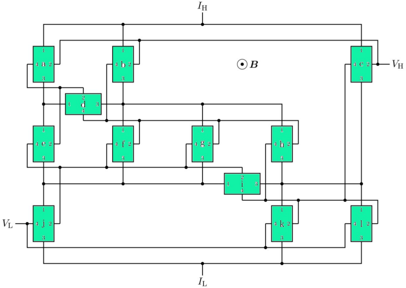


Each plateau i is centered on a resistance value $R_{\mathrm{H}} = R_{\mathrm{K}}/i$, with i integer

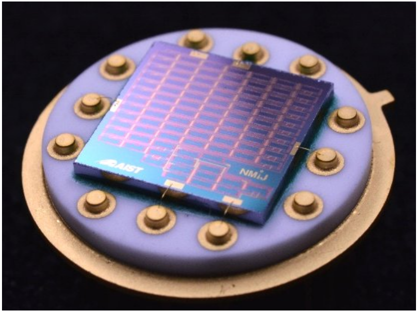
$$R_{\mathrm{K}} = \frac{h}{e^2} = \frac{\mu_0\, c}{2\alpha}.$$

R_{K} is linked to the fine structure constant α which can be measured by non-electrical means.

Quantum Hall array resistance standards



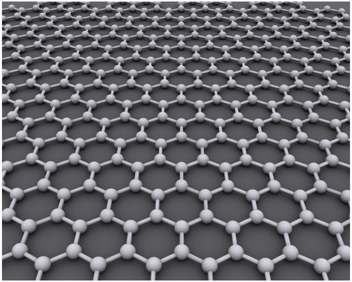
(a) 10 kΩ QHARS design (Ortolano et al., 2015)

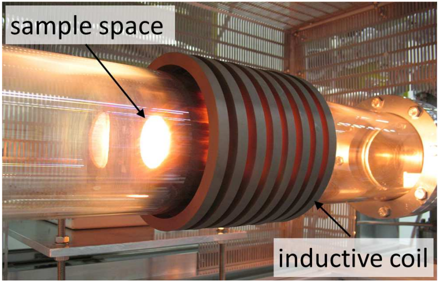


(b) 1 MΩ QHARS (Oe et al., 2016)

$$10\,\mathrm{k\Omega}\ \text{array:}\ R_{10\,\mathrm{k\Omega}} = \frac{203}{262} R_{\mathrm{H}} = (1 - 3.4 \times 10^{-8}) \times 10\,\mathrm{k\Omega}$$

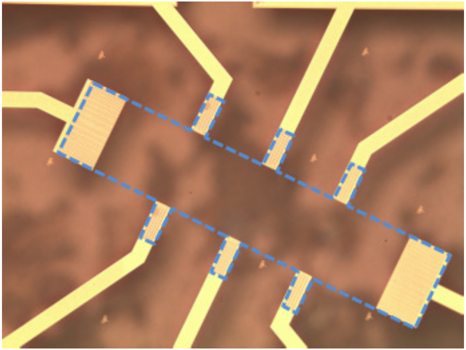
Graphene for QHE





Graphene for QHE

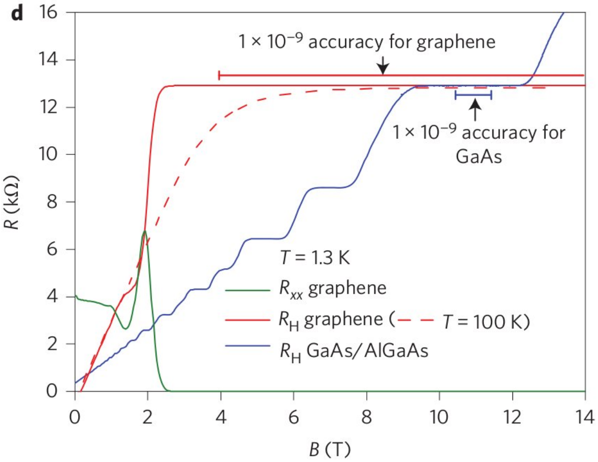
PTB graphene Hall bar



Courtesy: PTB

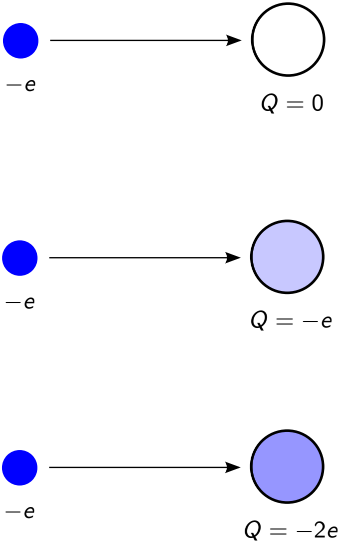
Graphene for QHE

(Ribeiro-Palau et al., 2015)



Quantized charge counting

Single charge confinement



$$E = 0$$

$$V = 0$$

$$E = \frac{-e \cdot Q}{2C} = \frac{e^2}{2C}$$

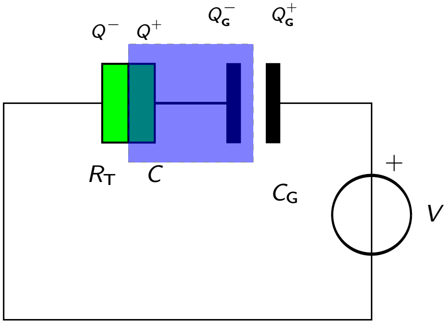
$$V = \frac{-e}{2C}$$

$$E = \frac{-e \cdot Q}{2C} = \frac{2e^2}{2C}$$

$$V = \frac{-2e}{2C}$$

Quantized charge counting

Single charge confinement



Single-electron box, coupled to an external circuit with a tunnel junction (with tunnel resistance R_T and capacitance C) and a capacitor C_G.

Quantized charge counting

Moving individual electrons

Via tunneling events, electrons charge the island with charge $Q_\mathrm{i} = -n\,e$, where n is an integer and e the charge quantum. The gate has capacitance C_G and holds charge Q_G; the tunnel junction has tunnel resistance R_T, capacitance C and holds charge Q; then, $Q_\mathrm{i} = Q - Q_\mathrm{G}$.

Circuit analysis of the mesh gives

$$E = \frac{Q^2}{2C} + \frac{Q_\mathrm{G}^2}{2C_\mathrm{G}} = \frac{C\,C_\mathrm{G}\,V^2 + Q_\mathrm{i}^2}{2(C + C_\mathrm{G})},$$

the generator work $W = Q_\mathrm{G}\,V$, and the free energy F

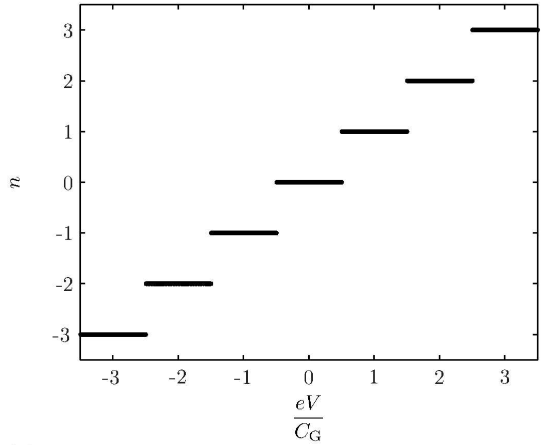
$$F = E - W = \frac{(C_\mathrm{G}\,V + Q_\mathrm{i})^2}{C + C_\mathrm{G}} + K = \frac{(C_\mathrm{G}\,V - n\,e)^2}{C + C_\mathrm{G}} + K$$

can be computed (K is a constant term).

Quantized charge counting

Moving individual electrons

At equilibrium at a given bias V, the minimization of the free energy $F(V)$ gives the corresponding equilibrium electron occupation of the box $n(V)$



Single-electron box occupation number n versus applied bias voltage V.

Quantized charge counting

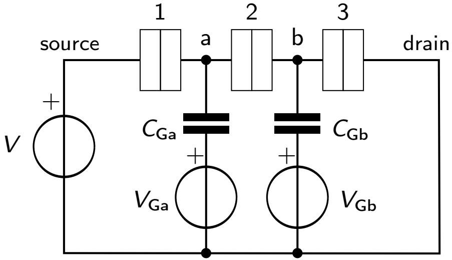
Conditions

In the derivation above, two hypoteses have been made:

1. the spacing between energy levels of the single electron box is large with respect to the average thermal excitation: $\dfrac{e^2}{2(C + C_\mathrm{G})} \gg k_\mathrm{B}\Theta$. With nanofabrication techniques, device capacitances in the fF range can be achieved; an adequate working temperature lies in the tens of mK range.
2. the spacing between energy levels of the single electron box is large with respect to the energy uncertainty of an occupation state, in turn related by uncertainty principle to the state lifetime $R_\mathrm{T}C$ caused by tunneling events. This gives the condition $R_\mathrm{T} \gg R_\mathrm{K}$.

Quantized charge counting

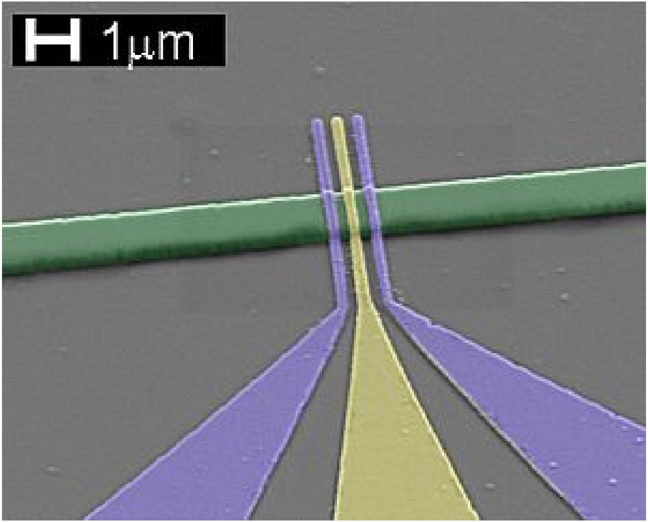
Nanodevices



A three-junction single-electron pump.

Quantized charge counting

Nanodevices

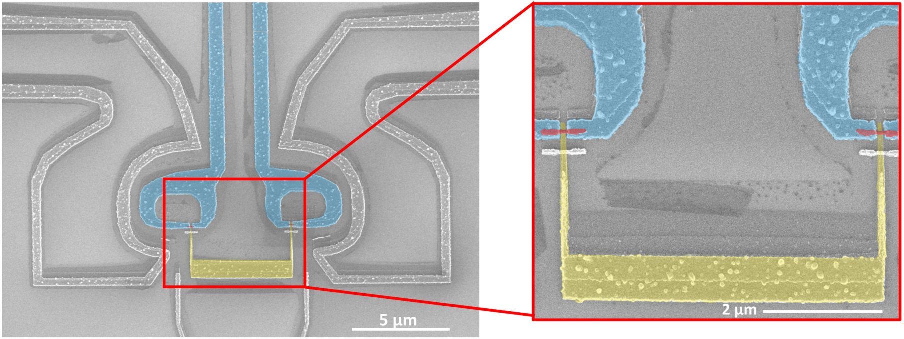


Courtesy: PTB

Semiconductor single-electron pump.

Quantized charge counting

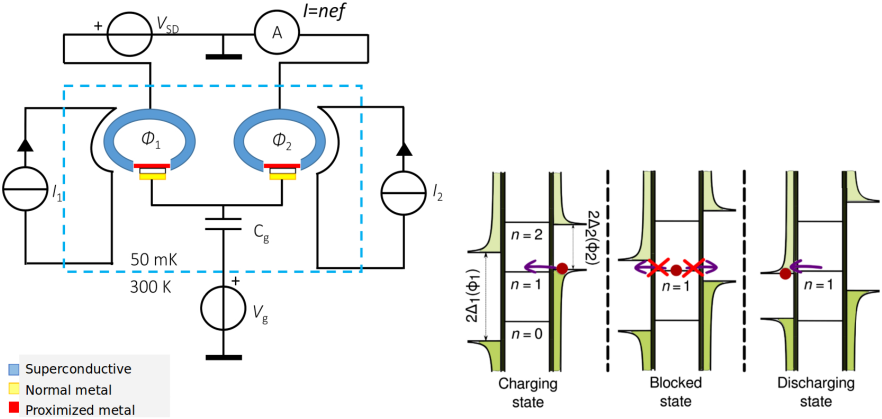
Superconducting quantum-interference single electron transistor (SQUISET)



Courtesy: Emanuele Enrico, INRIM

Quantized charge counting

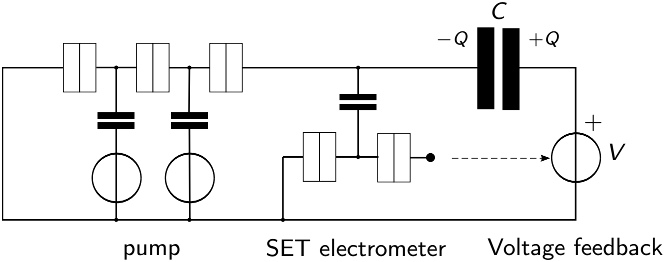
Superconducting quantum-interference single electron transistor (SQUISET)



Courtesy: Emanuele Enrico, INRIM

The Electron-counting capacitance standard

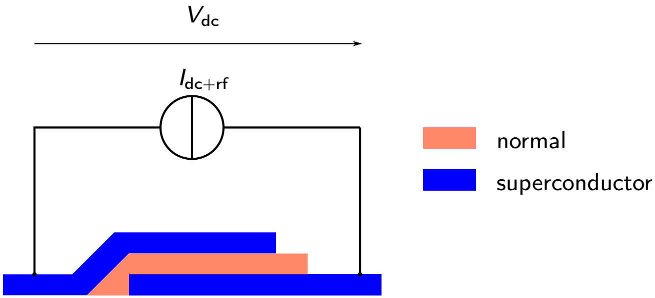
The definition of capacitance $Q = C\,V$ is directly employed in dc regime.



Electron-counting capacitance standards block schematics, using single-electron devices. A single-electron pump charges capacitor C to Q; a SET electrometer nulls low voltage side of C by driving a feedback generator to voltage V.

Counting flux quanta

Josephson junctions



Josephson junction:

- two superconductors coupled by a tunneling barrier
- have coupled wavefunctions

Counting flux quanta

The Josephson effect

$$i(t) = I_\mathrm{c} \sin\left(2\pi \frac{\phi(t)}{\Phi_0}\right)$$

where

$\Phi_0 = h/2e \approx 2.068\,\mathrm{Wb}$ is the flux quantum;

$K_\mathrm{J} = 2e/h = \Phi_0^{-1} \approx 483\,\mathrm{THz/V}$ is the Josephson constant;

I_c is the critical current of the junction;

$\phi(t) = \int_0^t v(\tau)\,\mathrm{d}\tau$ is the flux of the voltage applied to the junction.

Counting flux quanta

voltage to frequency converter: the AC Josephson effect

Applying a constant voltage V to the junction, $\phi(t) = Vt$,

$$i(t) = I_{\mathrm{c}} \cos\left(\frac{2\pi}{\Phi_0} Vt\right)$$

which is an oscillator with frequency $f = \dfrac{V}{\Phi_0}$

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

Applying a dc+ac voltage excitation $v(t) = V_{\mathrm{dc}} + V_{\mathrm{ac}} \cos(2\pi f_{\mathrm{ac}} t)$, the Josephson carrier $f_{\mathrm{J}} = V_{\mathrm{dc}}/\Phi_0$ is FM modulated.

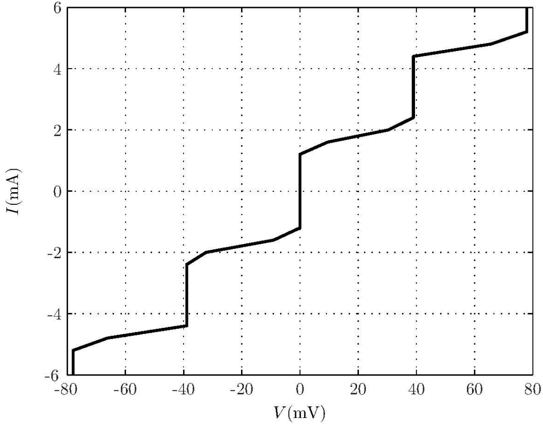
The FM sidebands allow a zero-frequency (dc) current bias only for the condition $f_{\mathrm{J}} = n f_{\mathrm{ac}}$, integer n:

$$V_{\mathrm{dc}} = n \Phi_0 f_{\mathrm{ac}} = \frac{n f_{\mathrm{ac}}}{K_{\mathrm{J}}}$$

Every cycle of f_{ac}, n flux quanta are counted across the junction.
Feasible drive frequencies: $f_{\mathrm{ac}} = 70\,\mathrm{GHz} \quad \Rightarrow \quad V_{\mathrm{dc}} = 150\,\mu\mathrm{V}$.

Counting flux quanta

frequency to voltage converter: the (inverse AC) Josephson effect

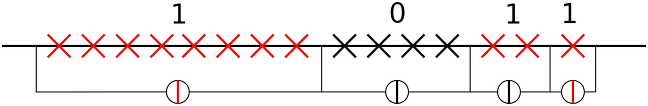


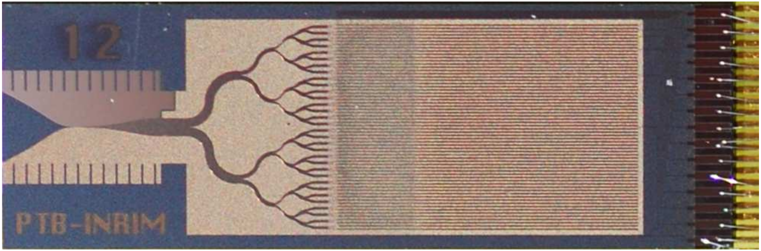
The $I - V$ characteristic of a Josephson array (256 junctions) under microwave irradiation. Steps $n = 0, \pm 1, \pm 2$ are visible. $f \approx 73\,\text{GHz}$

Counting flux quanta

Josephson binary DAC

Binary-weighted Josephson DAC



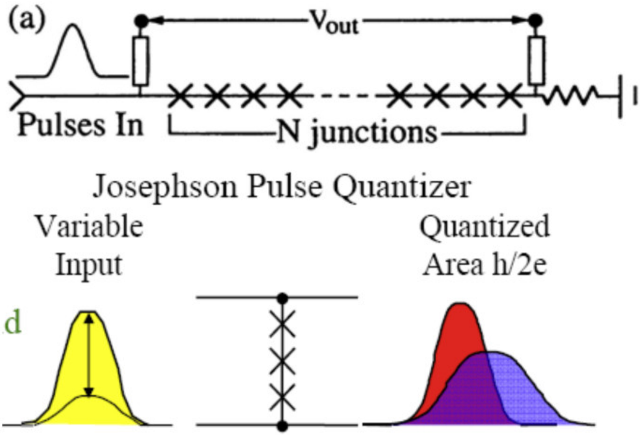


Josephson junction binary array chip. 13 bit+sign DAC with 8192 superconducting-normal metal-insulator-superconductor (SNIS) junctions. The junctions are geometrically arranged over 32 parallel strips of 256 junctions each. $f = 70\,\text{GHz}$. $V_{\text{fullscale}} \approx \pm 1.2\,\text{V}$

Counting flux quanta

Josephson pulsed DAC

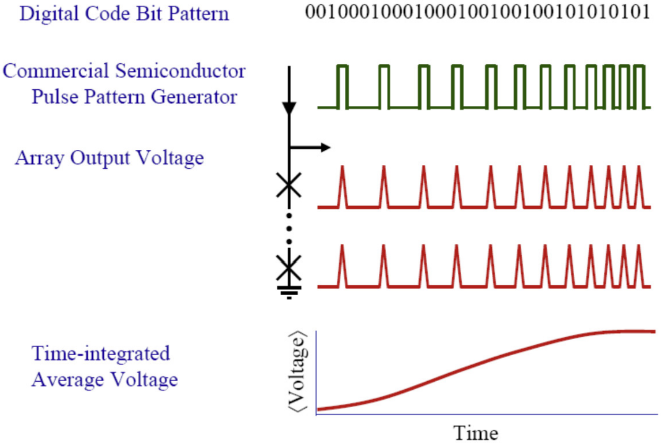
Quantization of RF pulses in quanta of area Φ_0



Counting flux quanta

Josephson Arbitrary Wafeform Generator (JAWS)

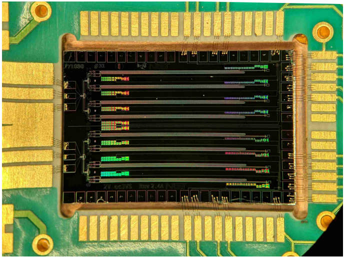
Pulse density modulation (PDM) allows to generate arbitrary waverforms



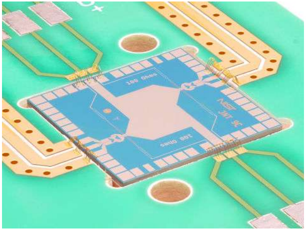
Counting flux quanta

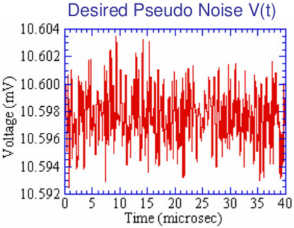
Applications of pulse density modulation:

Josephson arbitrary waveform syntesizer (JAWS)



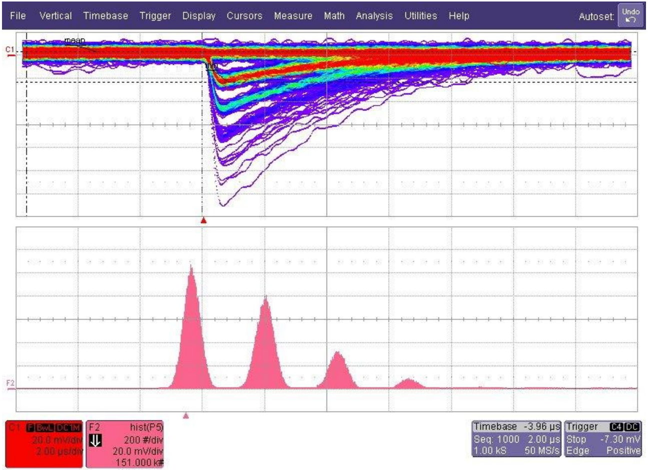
Pseudo-random noise reference for noise thermometry





. . . more counting: quantum candela

superconducting transition-edge sensors (TES) bolometers for single-photon counting at visible wavelengths)



C1 F BwL DC1M 20.0 mV/div 2.00 µs/div

F2 hist(P5) 200 #/div 20.0 mV/div 151.000 k#

Timebase -3.96 µs
Seq: 1000 2.00 µs
1.00 kS 50 MS/s

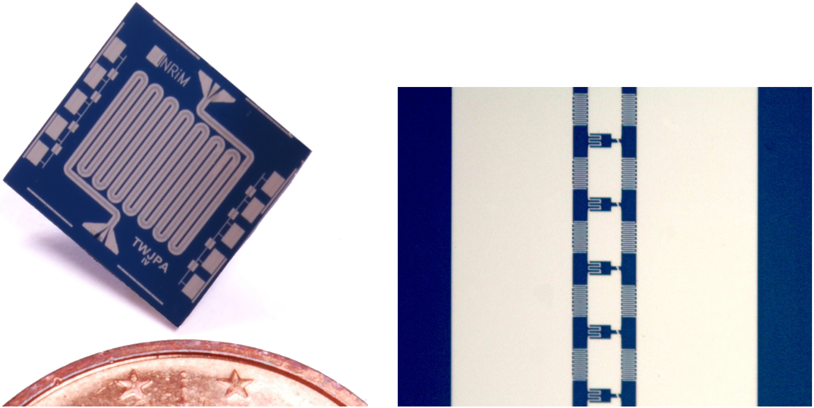
Trigger C4 DC
Stop -7.30 mV
Edge Positive

Courtesy: Mauro Rajteri, INRIM

Each photon: $3 \times 10^{-19}\,\mathrm{J} = 30\,\mathrm{aJ}$

. . . more counting: microwave photons

Travelling-wave Josephson parametric amplifier

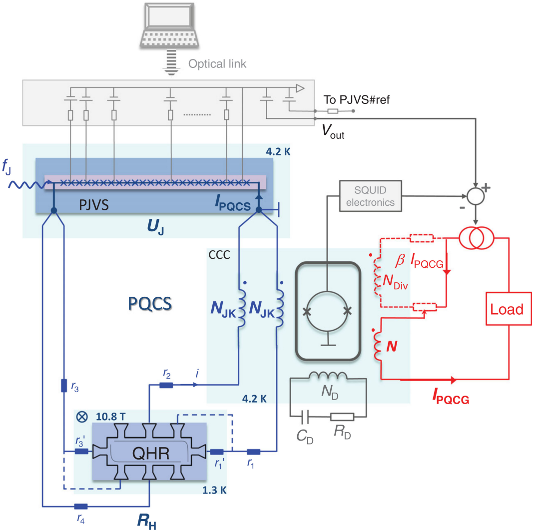


Courtesy: Emanuele Enrico, INRIM

Each photon: $10^{-24}\,\mathrm{J} = \mathrm{yJ}$

Quantum standards together

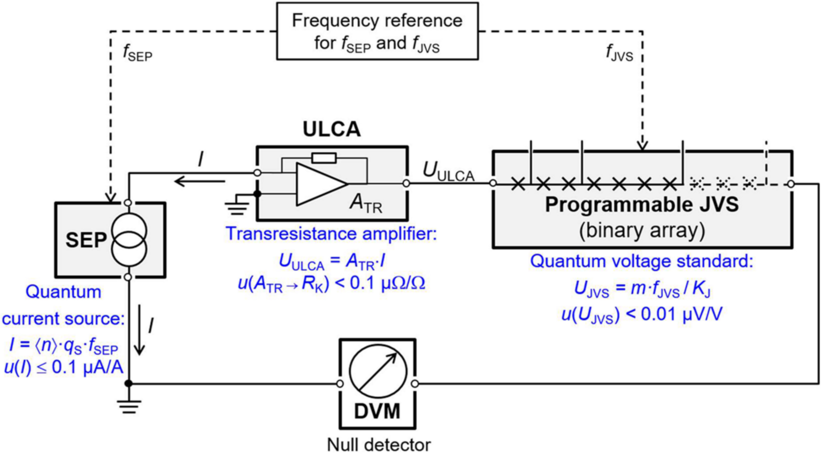
Josephson + QHE: dc current standard



(Brun-Picard et al., 2016, Fig. 2)

Quantum standards together

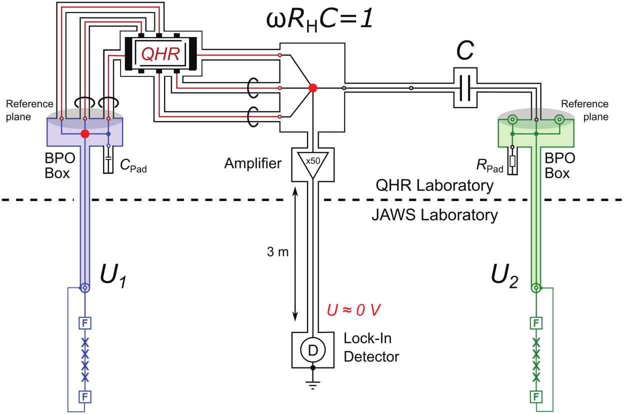
SET + QHE + Josephson: verification of SET pumps



(Scherer and Schumacher, 2019, Fig. 5)

Quantum standards together

JAWS + QHE: quantum impedance standard



(Bauer et al., 2017, Fig. 3)

`ptb.de/empir2019/giqs`

The SI 1960-2019: status of the quantum experiments

Knowledge in 1989 (CODATA):

$$K_\mathrm{J} = 483\,597.9(2)\,\mathrm{GHz/V} \qquad [4 \times 10^{-7}]$$

$$R_\mathrm{K} = 25\,812.807(5)\,\Omega \qquad [2 \times 10^{-7}]$$

but, *reproducibility* of Josephson and quantum Hall experiments in different experiments and different laboratories was much higher: 10^{-9}–10^{-10}

Solution: invent non-SI units! 18th CGPM resolution 6: Valid since January 1, 1990:

$$K_\text{J-90} = 483\,597.9\,\mathrm{GHz/V} \qquad [\text{exact}]$$

$$R_\text{K-90} = 25\,812.807\,\Omega \qquad [\text{exact}]$$

To $K_\text{J-90}$ and $R_\text{K-90}$ the conventional units Ω_{90}, H_{90}, F_{90}, A_{90}, W_{90} are associated.

These are the electrical units in use until 2019.

The Kibble balance

Determination of the Planck constant

Now the derivation can be clarified

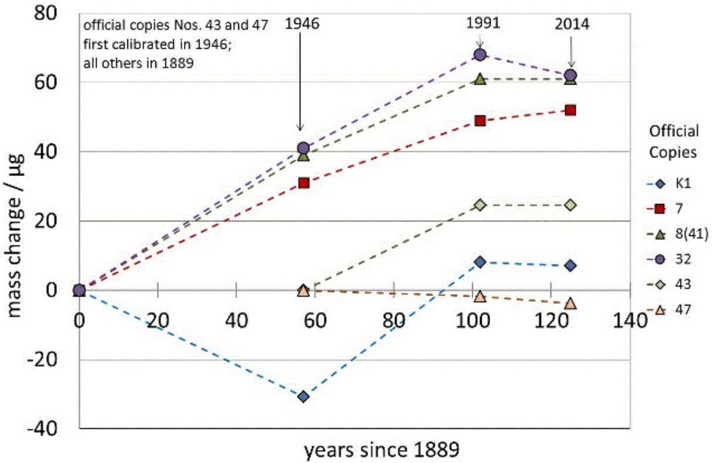
- $mgv = EI$
- $E = n\dfrac{f_\mathrm{E}}{K_\mathrm{J}}$
- $I = \dfrac{V_\mathrm{I}}{R} = \dfrac{f_\mathrm{I}}{K_\mathrm{J}}\dfrac{1}{rR_\mathrm{K}}$
- $K_\mathrm{J} = \dfrac{2e}{h}$
- $R_\mathrm{K} = \dfrac{h}{e^2}$

$$\Rightarrow mgv = hf_\mathrm{E}f_\mathrm{I}\frac{n}{r}$$

h can be measured mechanically

The SI, 1960-2019 : Problems

Problem: The drift of the International Prototype



The International Prototype Kilogram compared with its *témoins*
IPK might have lost 35 µg over 130 years

Problem: The SI and conventional units

Two incompatible systems

Becuase of improvements in the measurement of fundamental constants, the conventional and SI units started to drift apart. For example, CODATA 2014:

$$K_\mathrm{J} = 483\,597.8525(30)\,\mathrm{GHz/V} \qquad [6.1 \times 10^{-9}]$$

$$R_\mathrm{K} = 25\,812.807\,455\,5(59)\,\Omega \qquad [2.3 \times 10^{-10}]$$

Therefore

$$V_{90} = 1 + 9.8(6) \times 10^{-8}\,\mathrm{V}$$

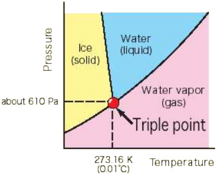
$$\Omega_{90} = 1 - 1.764(2) \times 10^{-8}\,\Omega$$

⇒ Unacceptable deviation of the conventional units respect to the SI units

Problem: uniformity of unit definitions

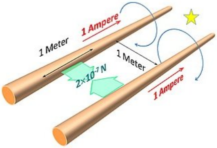
an **artefact**:

> *The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.*



a **natural property**

> *The kelvin is the fraction* 1/273.16 *of the thermodynamic temperature of the triple point of water.*



an **idealized experiment**

> *The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length [...] would produce a force equal to* 2×10^{-7} *newton per metre of length*

Two decades of discussions . . .

INSTITUTE OF PHYSICS PUBLISHING METROLOGIA

Metrologia **42** (2005) 71–80 doi:10.1088/0026-1394/42/2/001

Redefinition of the kilogram: a decision whose time has come

Ian M Mills[1], Peter J Mohr[2], Terry J Quinn[3], Barry N Taylor[2] and Edwin R Williams[2]

The revision of the SI, 2019-

Formal decision: the CGPM

26th General Conference of Weights and Measures

26^{e} CGPM
Versailles
13-16 novembre 2018

Implementation day: May 20, 2019, the World Metrology Day

The revised SI, 2019-

Units and fundamental constants in the SI

Fundamental constants (e.g., c, e, h, k_B) are special quantities in that they are considered universal and immutable.

1960-2019 paradigm

Most base units are defined independently from the fundamental constants. If K is a fundamental constant and $[K]$ is its, possibly derived, unit, then the numerical value of K is determined through an experiment as

$$\{K\} = \frac{K}{[K]}.$$

Revised SI paradigm

All base units are defined so that seven fundamental constant have an exact numerical value. If K is a fundamental constant, its numerical value $\{K\}$ is set by definition and $[K]$ is indirectly obtained as

$$[K] = \frac{K}{\{K\}}.$$

The revised SI, 2019-

The seven base units

The SI is the system of units in which:

- s The unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{\mathrm{Cs}}$ is $9\,192\,631\,770\,\mathrm{Hz}$;
- m the speed of light in vacuum c is $299\,792\,458\,\mathrm{m/s}$;
- kg the Planck constant h is $6.626\,070\,15 \times 10^{-34}\,\mathrm{J\,s}$;
- A the elementary charge e is $1.602\,176\,634 \times 10^{-19}\,\mathrm{C}$;
- K the Boltzmann constant k is $1.380\,649 \times 10^{-23}\,\mathrm{J/K}$;
- mol the Avogadro constant N_{A} is $6.022\,140\,76 \times 10^{23}\,\mathrm{mol}^{-1}$;
- cd the luminous efficacy of monochromatic radiation of frequency $540 \times 10^{12}\,\mathrm{Hz}$, K_{cd}, is $683\,\mathrm{lm/W}$,

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, W, respectively, are related to the units second, metre, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, cd, respectively, according to $\mathrm{Hz} = \mathrm{s}^{-1}$, $\mathrm{J} = \mathrm{m}^2\mathrm{kg}\mathrm{s}^{-2}$, $\mathrm{C} = \mathrm{A\,s}$, $\mathrm{lm} = \mathrm{cd\,sr}$, $\mathrm{W} = \mathrm{m}^2\mathrm{kg\,s}^{-3}$.

The SI, 2019-: the base units kilogram and ampere

The kilogram:

> *The kilogram, symbol kg, is the SI unit of mass. It is defined by taking the fixed numerical value of the Planck constant h to be* $6.626\,070\,15 \times 10^{-34}$ *when expressed in the unit J s, which is equal to* $\mathrm{kg\,m^2\,s^{-1}}$*, where the metre and the second are defined in terms of c and* $\Delta\nu_{\mathrm{Cs}}$*.*

The ampere:

> *The ampere, symbol A, is the SI unit of electric current. It is defined by taking the fixed numerical value of the elementary charge e to be* $1.602\,176\,634 \times 10^{-19}$ *when expressed in the unit C, which is equal to A s, where the second is defined in terms of* $\Delta\nu_{\mathrm{Cs}}$*.*

The SI, 2019-: the base units kilogram and ampere

The kelvin:

> *The kelvin, symbol K, is the SI unit of thermodynamic temperature. It is defined by taking the fixed numerical value of the Boltzmann constant k to be* $1.380\,649 \times 10^{-23}$ *when expressed in the unit* JK^{-1}, *which is equal to* $kgm^2\ s^{-2}\ K^{-1}$, *where the kilogram, metre and second are defined in terms of h, c and* $\Delta\nu_{Cs}$.

The mole:

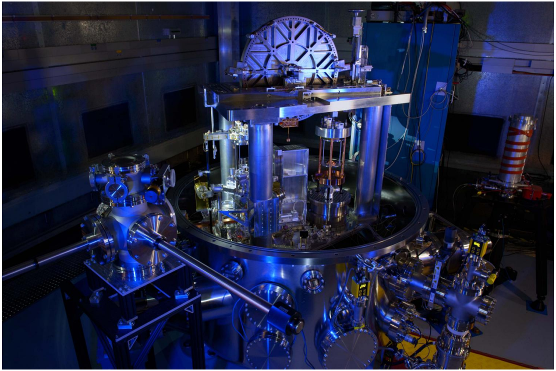
> *The mole The mole, symbol mol, is the SI unit of amount of substance. One mole contains* $6.022\,140\,76 \times 10^{23}$ *elementary entities. This number is the fixed numerical value of the Avogadro constant,* N_A, *when expressed in the unit* mol^{-1} *and is called the Avogadro number.*

The SI, 2019- : an electrical realization of the kilogram

The Kibble balance, revisited

h is **exact**;

$\Rightarrow$ The Kibble balance, if traceable to K_J and R_K, is a realization of the kilogram.



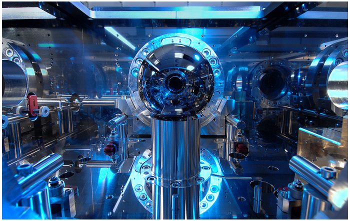
The SI, 2019- : a mechanical realization of the kilogram

Silcon atom counting

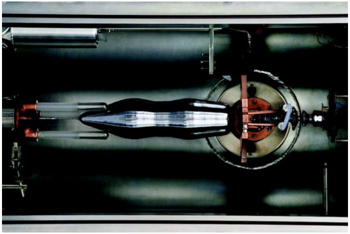
$$\begin{aligned} M_{\text{sphere}} &= N \cdot m_{\text{Si}} \\ &= \frac{V_{\text{sphere}}}{v_{\text{cell}}} m_{\text{Si}} \end{aligned}$$

Count the atoms

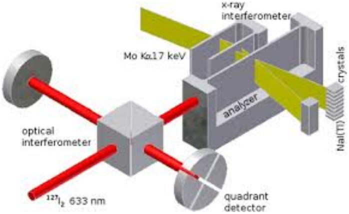
V_{sphere}: spherical interferometer



m_{Si}: single ^{28}Si crystal



v_{cell}: X-ray + optical interferometer



m_{Si}/h: known $[10^{-9}]$
from atomic experiments

$$M_{\text{sphere}} = \frac{V_{\text{sphere}}}{v_{\text{cell}}} \left(\frac{m_{\text{Si}}}{h} \right) h$$

And h is fixed in the new SI!

The SI, 2019- : a new status of quantum metrology

e has a fixed value **exact**;

$\Rightarrow$ any electron-counting experiment is a realization of the ampere;

$R_\mathrm{K} = \dfrac{h}{e^2}$ is **exact**;

$\Rightarrow$ the quantum Hall effect is a realization of the ohm;

$K_\mathrm{J} = \dfrac{2e}{h}$ is **exact**;

$\Rightarrow$ the Josephson effect is a realization of the volt;

$\Rightarrow$ The combined Josephson and quantum Hall effects, through Ohm's law, is a realization of the ampere.

The SI, 2019-

The electromagnetic constants

μ_0 the magnetic constant is not anymore $4\pi \times 10^{-7}$ H/m:
not exact and subject of measurement;

$\epsilon_0 = \dfrac{1}{\mu_0 c^2}$ the electric constant is no more exact;

$\Rightarrow$ ϵ_0 and μ_0 will have the same relative uncertainty
and will be totally correlated (correlation coefficient $= -1$)

$Z_0 = \mu_0 c$ the impedance of free space, and

$Y_0 = (\mu_0 c)^{-1}$ the admittance of free space are no more exact;

The SI, 2019-

The electromagnetic constants

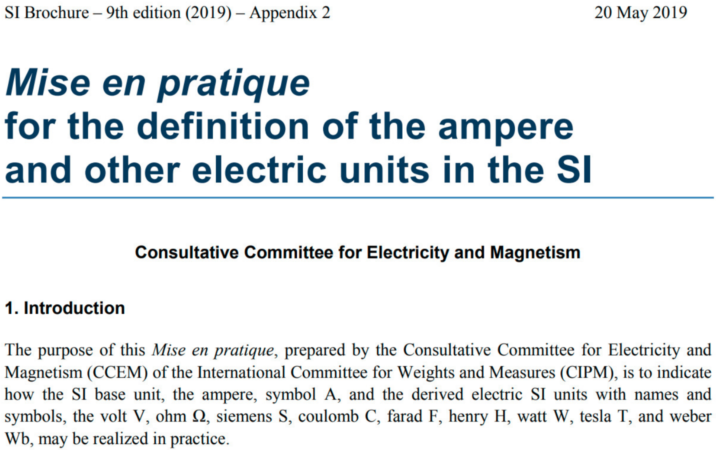
$$\alpha = \frac{e^2}{\epsilon_0 hc}$$
$$\alpha^{-1} = 2\frac{R_K}{Z_0} = 137.035\,999\,139(31)$$

is not exact, but can be measured with very high accuracy (2.3×10^{-10} CODATA 2014) via atomic spectroscopy experiments.

- μ_0 has the same uncertainty of α (2.3×10^{-10}),
- $\Rightarrow$ the calculable inductor keeps the status of a practical realization of the henry;
- $\Rightarrow$ the calculable capacitor keeps the status of a practical realization of the farad.

The SI, 2019-

Mise en pratique



SI Brochure – 9th edition (2019) – Appendix 2 20 May 2019

Mise en pratique for the definition of the ampere and other electric units in the SI

Consultative Committee for Electricity and Magnetism

1. Introduction

The purpose of this *Mise en pratique*, prepared by the Consultative Committee for Electricity and Magnetism (CCEM) of the International Committee for Weights and Measures (CIPM), is to indicate how the SI base unit, the ampere, symbol A, and the derived electric SI units with names and symbols, the volt V, ohm Ω, siemens S, coulomb C, farad F, henry H, watt W, tesla T, and weber Wb, may be realized in practice.

Electrical units: back within SI!

8/12/2017
Version 1.0

CCEM Guidelines for Implementation of the ‘Revised SI’

Consultative Committee for Electricity and Magnetism

- $V_{90} \Rightarrow V$: $d = +1.067 \times 10^{-7}$
- $\Omega_{90} \Rightarrow \Omega$: $d = +1.779 \times 10^{-8}$

What to do with maintained standards?

$d < 2.5\,U$: no action until next recalibration

$d > 2.5\,U$: numerical correction to be applied

The SI, 2019-

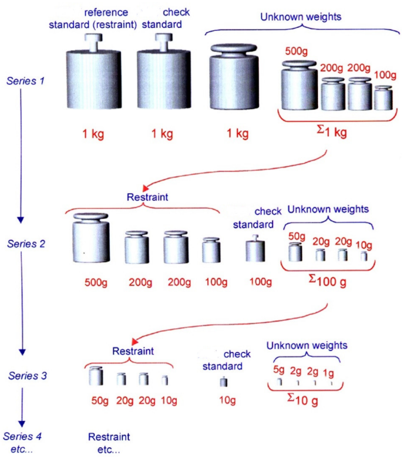
Unit definitions do not suggest preferred realisations;

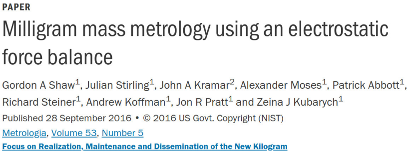
Any physical experiment that satisfies the definition is a realization of the unit

Units can be realized at any level (multiple or submultiple)

Any laboratory can realise the SI units at the uncertainty level of interest

Example: electrostatic realisation of the mg





PAPER

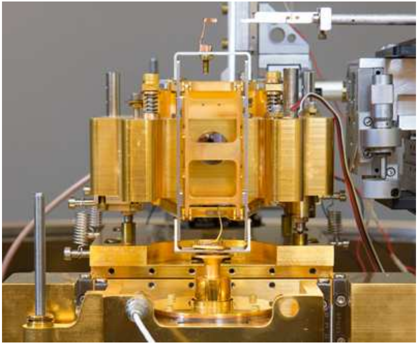
Milligram mass metrology using an electrostatic force balance

Gordon A Shaw[1], Julian Stirling[1], John A Kramar[2], Alexander Moses[1], Patrick Abbott[1], Richard Steiner[1], Andrew Koffman[1], Jon R Pratt[1] and Zeina J Kubarych[1]

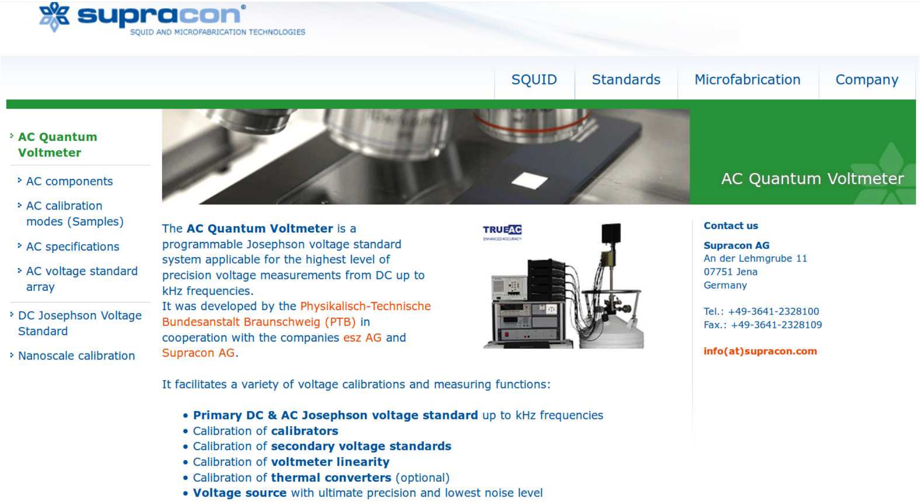
Published 28 September 2016 • © 2016 US Govt. Copyright (NIST)

Metrologia, Volume 53, Number 5

Focus on Realization, Maintenance and Dissemination of the New Kilogram



Example: a commercial quantum realisation of the volt



- **AC Quantum Voltmeter**
  - AC components
  - AC calibration modes (Samples)
  - AC specifications
  - AC voltage standard array
- DC Josephson Voltage Standard
- Nanoscale calibration

The **AC Quantum Voltmeter** is a programmable Josephson voltage standard system applicable for the highest level of precision voltage measurements from DC up to kHz frequencies.
It was developed by the Physikalisch-Technische Bundesanstalt Braunschweig (PTB) in cooperation with the companies esz AG and Supracon AG.

It facilitates a variety of voltage calibrations and measuring functions:

- **Primary DC & AC Josephson voltage standard** up to kHz frequencies
- Calibration of **calibrators**
- Calibration of **secondary voltage standards**
- Calibration of **voltmeter linearity**
- Calibration of **thermal converters** (optional)
- **Voltage source** with ultimate precision and lowest noise level

Contact us

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Example: an optical realisation of the pascal

nature > news > article

MENU

nature
International journal of science

METROLOGY

Pressure gets an upgrade

A 400-year-old method for measuring the quantity has a rival based on quantum physics.

BY ELIZABETH GIBNEY

Researchers in the United States have developed a new way to define and measure pressure and its unit, the pascal — one that they say will, within a year, begin to replace the mercury-based measurement methods that have been in use since 1643.

Pressure is conventionally defined as force per unit area, and the pascal is a force of 1 newton per metre squared. For nearly 400 years, values at air pressure and below have been measured using mercury-based instruments called manometers. The US National Institute of Standards and Technology (NIST) in Gaithersburg, Maryland, holds one of a handful of the world's most precise manometers, known as primary standards — huge instruments that serve as the benchmarks against which all other pressure sensors are calibrated. But NIST scientists have now developed a highly precise method for measuring pressure that is based on treating it as energy density. This is an equivalent physical description to force per unit area because it is derived from the same combination of 'base' units, the most fundamental units of measure in the International System of Units (SI).

The NIST method involves probing atoms of gas in a cavity directly with a laser to determine their pressure. The team hopes to show in the next year that its apparatus can rival the manometer — and to encourage other metrology labs to use it as their primary standard.

If widely accepted by the metrology community, the method would do away with the need for mercury, which is toxic and faces international bans. Moreover, the new technique allows metrologists to measure pressure directly, using a fundamental constant of nature, and does not rely on previous measurements of other quantities, such as density, on which the manometer depends. In theory, it could also allow anyone to measure pressure from first principles without "the tedious work of" a chain of calibrations to a primary standard that is currently required, says Bo Gao, a metrologist at the Technical Institute of Physics and Chemistry of the Chinese Academy of Sciences in Beijing, who works on a related method to measure low temperatures. The technique could enable faster measurements with more-portable equipment, benefiting industries such as aviation and semiconductor manufacturing.

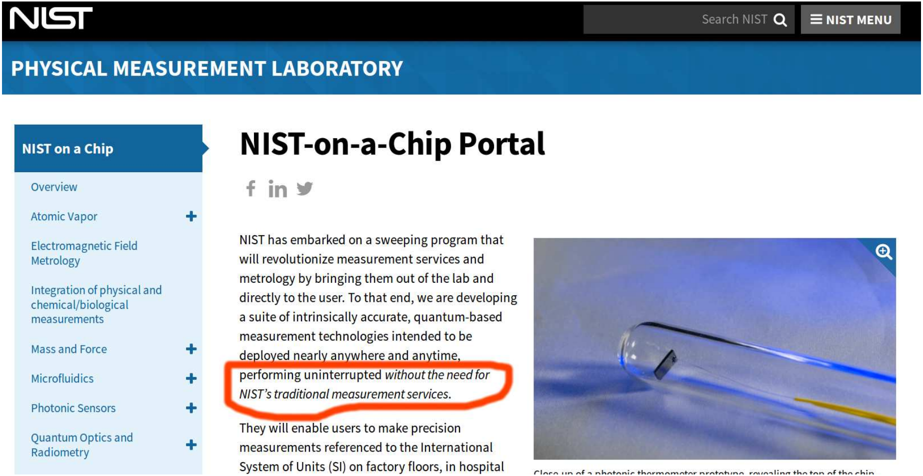
Metrologists have long wanted to replace manometers, the principles of which date back to the mercury pressure gauge invented by Italian physicist Evangelista Torricelli in 1643. Modern manometers have two tall columns of mercury, and measure the force exerted on a surface due to a pressure by balancing it against the force generated by the weight of mercury.

NIST

The FLOC measures gas pressure using lasers.

A new role for the national metrology institutes?

NIST-on-a-chip initiative



PHYSICAL MEASUREMENT LABORATORY

NIST-on-a-Chip Portal

NIST has embarked on a sweeping program that will revolutionize measurement services and metrology by bringing them out of the lab and directly to the user. To that end, we are developing a suite of intrinsically accurate, quantum-based measurement technologies intended to be deployed nearly anywhere and anytime, performing uninterrupted *without the need for NIST's traditional measurement services.*

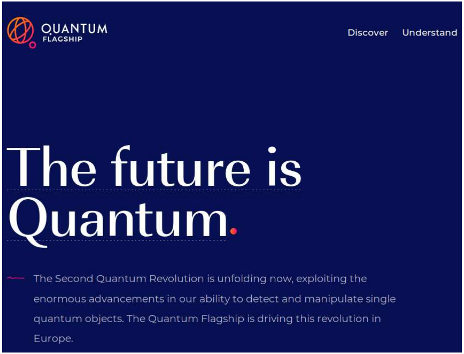
They will enable users to make precision measurements referenced to the International System of Units (SI) on factory floors, in hospital

Close up of a photonic thermometer prototype, revealing the top of the chip

and in Europe?

The quantum flagship, 1 b€ initiative

From the draft Strategic Research Agenda, pillar *Quantum metrology and sensing*:
“[. . .] application targets here are for enhanced measurement and metrology of current, resistance, voltage and magnetic fields [. . .] integration of quantum electrical standards for self-calibration in instrumentation providing highly-accurate measurements [. . .]”

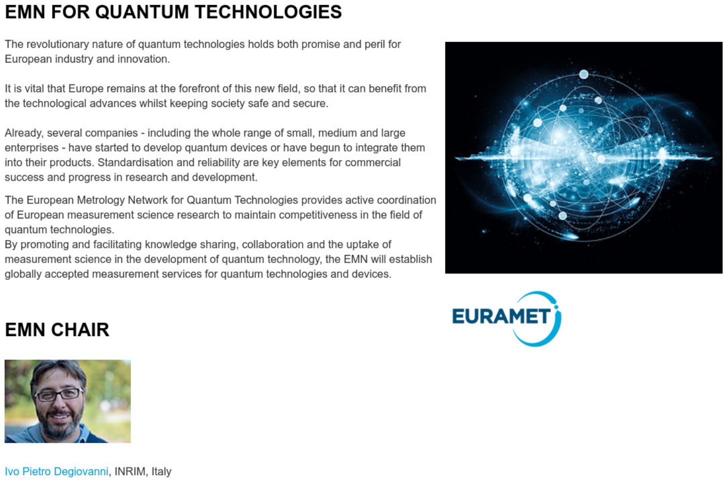


The future is Quantum.

The Second Quantum Revolution is unfolding now, exploiting the enormous advancements in our ability to detect and manipulate single quantum objects. The Quantum Flagship is driving this revolution in Europe.

and in Europe?

The European Metrology Network on Quantum Technologies



EMN FOR QUANTUM TECHNOLOGIES

The revolutionary nature of quantum technologies holds both promise and peril for European industry and innovation.

It is vital that Europe remains at the forefront of this new field, so that it can benefit from the technological advances whilst keeping society safe and secure.

Already, several companies - including the whole range of small, medium and large enterprises - have started to develop quantum devices or have begun to integrate them into their products. Standardisation and reliability are key elements for commercial success and progress in research and development.

The European Metrology Network for Quantum Technologies provides active coordination of European measurement science research to maintain competitiveness in the field of quantum technologies.
By promoting and facilitating knowledge sharing, collaboration and the uptake of measurement science in the development of quantum technology, the EMN will establish globally accepted measurement services for quantum technologies and devices.

EMN CHAIR

Ivo Pietro Degiovanni, INRIM, Italy

Thank you!

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